Constants of Nature, typical units of measure, and physical & astronomical formulae *(14 Aug. 2019):*

Some constants of Nature:

$G = 6.67 \times 10^{-8}$ c	*Newton's gravitational constant*	Units: $[cm^3\ s^{-2}\ g^{-1}]$
$c = 2.998 \times 10^{10}$	*Speed of light in vacuum*	$[cm\ s^{-1}]$
$h = 6.626 \times 10^{-27}$	*Planck's constant*	$[cm^2\ s^{-1}\ g]$
$e = 4.8 \times 10^{-10}$	*Electric charge of an electron or proton*	$[g^{1/2}\ cm^{3/2}\ s^{-1}]$ = [electrostatic units]
$k = 1.38 \times 10^{-16}$	*Boltzman's constant*	$[erg\ K^{-1}] = [g\ cm^2\ s^{-2}\ K^{-1}]$
$\sigma = 5.67 \times 10^{-5}$	*Stefan-Boltzmann constant*	$[erg\ s^{-1}\ cm^{-2}\ K^{-4}]$
$H = 71$ km s^{-1} Mpc^{-1}	*Hubble "constant"* $[s^{-1}]$ ~ $2.3\times10^{-18}\ s^{-1}$ ~ 1 /[Age of Universe] = $1/13.7\times10^9$ yrs]	
$\sigma_T = 6.652 \times 10^{-25}\ cm^2$	*Thompson cross-section of the electron*	

Some units of time:

1 year ~ [365.25][24][60][60] ~ 3.15×10^7 seconds   Age of the Universe: t_U~ 13.7×10^9 years

Planck time $\boldsymbol{\tau_P = (hG/2\pi c^5)^{0.5}}$~ 5.4×10^{-44} seconds *~ time to cross Planck length at the speed of light*

Some units of length:

1 A.U. = "**Astronomical Unit**" = $1.49597871 \times 10^{13}$ cm ~ $\mathbf{1.5 \times 10^{13}}$ **cm**: *Mean Earth–Sun separation.*

1 pc = 1 "**parsec**" = $\mathbf{3.086 \times 10^{18}}$ **cm (*Distance from which 1 A.U. subtends 1")***

1 kpc = $\mathbf{10^3}$ pc = $\mathbf{3.086 \times 10^{21}}$ cm:

1 Mpc = $\mathbf{10^6}$ pc = $\mathbf{3.086 \times 10^{24}}$ cm

1 μm (micro-meter) = $\mathbf{10^{-6}}$ m = $\mathbf{10^{-4}}$ cm	*~ 2x wavelength of visual light*
1 nm (nano-meter) = 10^{-9} m = 10^{-7} cm	*~ size of molecules*
1 Angstrom = 1 A = $\mathbf{10^{-8}}$ cm	*~ size of atoms*
1 Fermi = 10^{-13} cm	*~ size of atomic nuclei*
$R_o = 6.956 \times 10^{10}$ cm	*~ radius of the Sun*

Horizon radius of the Universe today: R_U ~ 1.3×10^{28} cm ~ *size of cosmic horizon (* $R_U \sim ct_{Universe}$*)*

Planck length = $\boldsymbol{\lambda_P = (hG/2\pi c^3)^{0.5}}$~ 1.6×10^{-33} cm *~ wavelength of photon which would collapse into its own black hole. Scale of the "Big Bang", of "String Theory", and black hole singularities.*

Some measures of Angle:

A circle contains **$360^o = 2\pi$ [radians] (rad)**

Radian: **1 radian** = 360 / 2 π [degrees] ~ **57.3^o**

Arc-minute: **1' = 1/60 of a degree**;

Arc-second: **1" = 1/60 of an arc-minute** = 1/3600 of a degree ~ 1 / 206265 or a radian

Thus 1 radian contains ~**206,265** arc-seconds ~ 57.3 x 3600 arc-seconds

Some units of energy:

1 Watt = $\mathbf{10^7}$ ergs $\mathbf{s^{-1}}$

1 electron Volt = **1 eV = $\mathbf{1.602 \times 10^{-12}}$** [erg]   Units of energy: [erg] = $[g\ cm^2\ s^{-2}]$

Kinetic energy of a particle	$\mathbf{E = \frac{1}{2} m V^2}$	
Energy of a photon	$\mathbf{E = h\nu}$	ν = frequency in Hz
Energy – mass relation	$\mathbf{E = \gamma m_o c^2}$	$\mathbf{m_o}$ = rest mass => $m = E / c^2$
Thermal energy	$\mathbf{E = kT}$	**T** = temperature in Kelvin (degrees above absolute 0, -273K)

1 megaton explosion $\mathbf{E_{mt} = 4.184 \times 10^{22}}$ [erg] :   A typical supernova E_{SN} ~ 10^{51} ergs

Some units of mass and luminosity:

Planck mass : $\mathbf{m_P} = \mathbf{(hc/2\pi G)^{0.5}}$~ 2.2×10^{-5} grams ~ *mass-energy of an EM wave with* $\boldsymbol{\lambda}$ *= Planck length*

Mass of a proton: $\mathbf{m_H} = \mathbf{m_p = 1.67 \times 10^{-24}}$ **grams** ; Mass of an electron $\mathbf{m_e = 0.9 \times 10^{-27}}$ **grams**

Mass of the Sun: $\mathbf{M_o = 1.989 \times 10^{33}}$ **grams** ; Luminosity of the Sun $\mathbf{L_o = 3.839 \times 10^{33}}$ **(erg $\mathbf{s^{-1}}$)**

Earth: $M_E = 5.9736 \times 10^{27}$ **grams**, $R_E = 6.378 \times 10^8$ **cm**

Jupiter: $M_J = 1.8986 \times 10^{30}$ **grams** ($\sim 10^{-3} M_o$), $R_J \sim 7 \times 10^9$ **cm** $= 7 \times 10^4$ **km**

Luminosity: $\mathbf{L = 4 \pi R^2 \sigma T^4}$ (erg s^{-1}) where **T** is in Kelvin, R is the radius of the radiating surface.

Flux $\mathbf{F = L / (4 \pi D^2)}$ (erg s^{-1} cm^{-2}) **D** is distance from the source to where the flux is measured.

Some measures of Geometry:

Circumference of a circle $\mathbf{C = 2 \pi r}$ where r is the radius of the circle

Area of a circle, $\mathbf{A = \pi r^2}$: Area of a sphere, $\mathbf{A = 4 \pi r^2}$: Volume of a sphere, $\mathbf{A = (4/3) \pi r^3}$

Velocity and acceleration:

Velocity **V** = [change in position] / [time interval] = $\mathbf{\Delta x / \Delta t}$

Acceleration **a** = [change in velocity] / [time interval] = $\mathbf{\Delta V / \Delta t = \Delta x / \Delta t^2}$

Forces: Force on particle of mass m **F = ma** **a** = acceleration [g cm s^{-2}]

Gravitational force $\mathbf{F = -G\, m\, M / r^2}$ *between masses m and M separated by distance* ***r***

Electrostatic force $\mathbf{F_E = e_1 e_2 / r^2}$ *between charges* $\boldsymbol{e_1}$ *and* $\boldsymbol{e_2}$ *separated by distance* ***r***

Magnetic force $\mathbf{F_B = e_1 V x B / c}$ $\mathbf{F_{EM} = F_E + F_B}$ *V = velocity*

Strong Nuclear Force short range (10^{-13} cm) *binds quarks into neutrons & protons*

Weak Nuclear Force short range (10^{-15} cm) *responsible for radioactivity and quark decays*

Centrifugal (centripedal) force $\mathbf{F_c = mV^2 / R}$; **V** = orbit speed, **R** = orbit radius

Power radiated by a charge **q**, experiencing acceleration **a**: $\mathbf{P = 2q^2a^2 / 3c^3}$ (erg s^{-1}) (Larmor)

Gravity & Orbits:

Circular orbit speed $\mathbf{V_{orbit} = (G\, M / r)^{1/2}}$

Escape speed $\mathbf{V_{escape} = (2\, G\, M / r)^{1/2}} = 2^{1/2} V_{orbit} \sim 1.414\ V_{orbit}$

Gravitational potential energy per gram, $\mathbf{E_G = GM / r}$; Self energy $\mathbf{E_G \sim GM^2 / r}$

Gravitational collapse time of a cloud with mean density ρ: $\boldsymbol{\tau_{coll} \sim 1/ (G\, \rho)^{0.5}}$ [sec]

Accretion rate from a collapsing isothermal sphere [$\rho(r) = \rho_0 r^{-2}$]: $\mathbf{dM /dt \sim c_s^3 / G}$ [g s^{-1}]

Speed of Sound: $\mathbf{c_s = (kT / \mu m_H)^{1/2}}$

Accretion luminosity produced by accretion rate dM/dt onto an object of mass M, radius r:

$\mathbf{L \sim GM(dM/dt) / r}$ [erg s^{-1}] ; Kelvin-Helmholtz time scale: $\boldsymbol{\tau_{KH} \sim GM^2 / RL}$ [sec]

Virial Theorem: **2 <[Kinetic Energy]> = - <[time averaged potential energy] >** ; $\mathbf{[mV^2] = [U]}$

<u>Elliptical orbits & binaries:</u> Ellipse with semi-major & semi-minor axes **a**, **b**, eccentricity **e**,

Radial distance from the foci, **r, r'**: **2a = r + r'** ; $\mathbf{b^2 = a^2 (1 - e^2)}$; $\mathbf{P^2 = a^3}$; $\mathbf{L = \mu\, V x\, r}$

$\mathbf{e = \Delta x / 2a}$ where $\mathbf{\Delta x}$ = separation between foci. Reduced mass: $\boldsymbol{\mu = m_1 m_2 / (m_1 + m_2)}$

$\mathbf{r = a(1 - e^2) / [1 + e \cos(\theta)] = (L^2 / \mu^2)/[GM(1 + e \cos(\theta)]}$; $\mathbf{L = \mu\ [GMa(1 - e^2)]^{0.5}}$ (Kepler I)

A = Area swept-out; $\mathbf{dA/dt = L / 2\mu}$; $\mathbf{V^2_{orbit} = G\, (m_1 + m_2)\, [(2/r) - (1/a)]}$ (Kepler II)

Orbit Period: $\mathbf{P^2 = 4 \pi^2 a^3 / [G\ (m_1 + m_2)]}$; Orbit energy: $\mathbf{E = -Gm_1m_2 / 2a}$ (Kepler III)\

Wavelengths of light & matter particles:

Speed of light $\mathbf{c = \nu \lambda}$ λ = *wavelength; f = frequency*

Wavelength of light $\mathbf{\lambda = c / \nu}$

Wavelength of a particle $\mathbf{\lambda = h / mV = h / p}$ (de Broglie wavelength): V = *particle velocity,*

p = mV = *momentum of a particle mass m;* $\mathbf{p = E/c = h\nu / c = h/ \lambda}$ = momentum of light

Telescopes and resolution: Angular resolution of a telescope:

$\mathbf{\theta = 1.22\, \lambda / D}$ [radians] : **D** = mirror or lens diameter; λ = wavelength

Magnification: $\mathbf{M = FL / f_e}$ = **D(entrance-pupil) / D(exit-pupil)** = **Angle(out) / Angle(in)**

$\mathbf{f_e}$= focal length of the eyepiece; **FL** = effective focal length of the main mirror or lens

Effective aperture: $\mathbf{A_{eff} = \eta \pi\, (D/2)^2}$ where $\boldsymbol{\eta}$=`throughput efficiency' ~ 0.1 to 0.8 typically.

Magnitudes & Flux:

Apparent Magnitude: $\mathbf{m = -2.5 \log_{10} [Flux(\lambda) / Flux_{Vega}(\lambda)]}$ (Vega scale)

Absolute magnitude: What the ***Apparent magnitude*** would be at ***D = 10 pc***. $\mathbf{M_V(Sun) = 4.74}$

1 Jansky = 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

Flux of **Vega** (m = 0 mag.): **3781 Jy at λ = 0.55 μm** (visual Cousins-Johnson **V-filter**)

For conversions of **m** to **Jy** in other filters, see …

http://ssc.spitzer.caltech.edu/warmmission/propkit/pet/magtojy/

Non-relativistic Doppler Effect: $\mathbf{\Delta\lambda/\lambda = \Delta f/f = \Delta V / c = z}$ (= redshift if ΔV is positive) where

$\Delta\lambda = \lambda_{observed} - \lambda_{emitted}$; $-\Delta f = f_{observed} - f_{emitted}$; $\Delta V = V_{observed} - V_{rest}$

Redshift or Blueshift: $z = \Delta\lambda / \lambda = [\lambda_{observed} - \lambda_{emitted}] / \lambda_{emitted} = [f_{emitted} - f_{observed}] / f_{observed} = \Delta f / f$

Relativistic Doppler Effect: $\mathbf{V = c\,[(\{1+z\}^2 - 1) / (\{1+z\}^2 + 1)]}$ where $z = \Delta\lambda/\lambda$ = redshift

Lorrentz transformations:

$\mathbf{x' = \gamma\,(x - Vt)}$; $\mathbf{v' = y'}$; $\mathbf{x' = z}$, $\mathbf{t' = \gamma(t - Vx/c^2)}$ with $\mathbf{\beta = v/c}$ & $\mathbf{\gamma = (1 - \beta^2)^{-1/2}}$

Time-dilation: $\Delta t_{moving} = \mathbf{(1 - \beta^2)^{-1/2}}\,\Delta t_{rest}$ Space-contraction: $\mathbf{L_{moving} = L_{rest}\,(1-\beta^2)^{1/2}}$

Black Holes, Cosmology, Heisenberg uncertainty:

$\mathbf{R_s = 2\,GM / c^2}$ - Radius of a black hole of mass M

V = H D - Hubble's Law; (D in Mpc, V in km/s)

H ~ 71 km s^{-1} / Mpc - H is the "Hubble constant"

$\mathbf{\Delta p = h / \Delta x}$ $\mathbf{\Delta E = h / \Delta t}$ - Heisenberg uncertainty principle.

Fundamental particles **("fermions"):** "Can't put two or more in the same place – like cars!"

Quarks: Up (**u** charge = +2/3 e); *Down* (**d** charge = -1/3 e) :**Stable in protons & neutrons**

Charmed (**c** charge = +2/3 e); *Strange* (**s** charge = -1/3 e) :Unstable (via Weak force)

Top (**t** charge = +2/3 e); *Bottom* (**b** charge = -1/3 e) :Very unstable (via Weak force)

Leptons: ***Electron*** (**e** charge = -e); *e-neutrino* (ν_e charge = 0) :**e -stable**; $\nu_e\,\nu_m\,\nu_t$ -oscillate

Muon (μ charge = +2/3 e); *μ-neutrino* (ν_m charge =0) :Unstable (via Weak force)

Tau (τ charge = +2/3 e); *τ-neutrino* (ν_t charge =0) :Very unstable (via Weak force)

All above have anti-matter counterparts with opposite charge (12 fundamental particles known)

Force carriers **("bosons"):** "Two or more love to be in the same place – like photons in lasers!"

Photons (electro-magnetic force) W^+, W^-, Z_0 (weak nuclear force) 8 *Gluons* (strong nuclear force)

Gravitons (gravitational force) *Higgs particle*: (makes particles such as $W^{+/-}$ and Z_0 massive)

Neutrons *are made of 3 quarks:* $[u^{2/3}, d^{-1/3}, d^{-1/3}]$; ***Protons*** *are made of 3 quarks:* $[u^{2/3}, u^{2/3}, d^{-1/3}]$

Mesons are made of quark-antiquark pairs: e.g. pions: $\pi^+ = [u^{2/3}\,\underline{d}^{+1/3}]$; $\pi^- = [\underline{u}^{-2/3}\,d^{-1/3}]$; $\pi^0 = [u^{2/3}\,\underline{u}^{-2/3}]$ &

$\pi^0 = [d^{1/3}\,\underline{d}^{-1/3}]$ (Pions decay. e.g. $\pi^+ \Rightarrow W^+ \Rightarrow \mu^+ + \nu_m$ followed by $\mu^+ \Rightarrow W^+ \Rightarrow e^+ + \nu_e$ etc.)

Black-body radiation, Larmor radiation formula, Cyclotron, Synchrotron, & Bremsstrahlung radiation:

Planck function: $\mathbf{B\nu(T) = [2\,h\nu^3 / c^2]\,\{1 / [\exp(h\nu/kT) - 1]\}}$ [erg s^{-1} cm^{-2} Hz^{-1} sr^{-1}]

Long-wave (Rayleigh-Jeans or RJ) limit:): $\mathbf{B\nu(T) = 2\,kT / \lambda^2 = 2\,kT\nu^2 / c^2}$ "

Short-wave limit: $\mathbf{B\nu(T) = [2\,h\nu^3 / c^2]\,\exp(-h\nu/kT)}$ "

Wavelength of the peak: $\mathbf{\lambda_{peak} = 0.29 / T(K)}$ [cm]

Larmor Radiation formula: Power radiated (erg s^{-1}) $\mathbf{P = (2/3)e^2a^2/c^3}$ a = acceleration ~V^2_{orbit} / r

Gyrofrequency (cyclotron radiation) $\mathbf{\omega_c = eB/mc}$ (radians s^{-1}) orbit time: $\mathbf{P_{orbit} = \omega/2\pi}$

Synchrotron radiation: beaming angle $\mathbf{\theta = \gamma^{-1}}$ Peak of spectrum at: $\mathbf{\omega_s = \gamma^2\,eB/mc}$

Spectral index, x: $\mathbf{S\nu = S_o\,\nu^x}$ Thermal (Rayleigh-Jeans) **x=2**, non-thermal, **x= negative**

Bremsstrahlung (free-free from a plasma): **x~2** at low ν; flat at higher ν (**x=-0.1** up to $h\nu \sim kT_{plasma}$)

Radiative Transfer: Light from background source with intensity or flux , $\mathbf{I_o(\nu)}$ passes through cloud with optical depth, $\tau(\nu)$, and emissivity, $\varepsilon(\nu)$. An element of optcial optical depth is given by $\mathbf{d\tau(\nu)=\kappa(\nu)\rho ds}$ where $\kappa(\nu)$ is the mass abruption coefficient.

The mean free path is $\lambda_{mfp} = 1 / n\,\sigma(\nu) = 1 / \kappa(\nu)\rho$

The observed intensity $I(\nu)$ is: $\mathbf{I(\nu) = I_o(\nu) \exp[-\tau(\nu)] + B(\nu,T)\{ 1 - \exp[-\tau(\nu)]\}}$

Optically thick ($\tau(\nu) >>1$): $\mathbf{I(\nu) = B(\nu,T)}$

Optically thin ($\tau(\nu) <<1$): $\mathbf{I(\nu) = I_o(\nu)\{1 -\tau(\nu)\} + \tau(\nu)B(\nu,T)}$

In terms of brightness temperature (or in Rayleigh-Jeans limit):

$\mathbf{T = T_o \exp[-\tau(\nu)] + T_{cloud}\{ 1 - \exp[-\tau(\nu)]\}}$ Thick: $\mathbf{T = T_{cloud}}$: Thin: $\mathbf{T = T_o[1 - \tau] + \tau\, T_{cl}}$

Collision rate: $\mathbf{R_{col} \sim n\sigma V}$ (s^{-1}) ; n = number density (cm^{-3}), σ=cross-section (cm^2), V=velocity (km s^{-1})

Collision rate per unit volume: $\mathbf{\mathcal{R}_{col} \sim n^2\sigma V}$ ($cm^{-3}\ s^{-1}$)

Einstein spontaneous decay rate: $A_{ul} = 64\pi^4\nu_{ul}^3\,\mu^2_{ul} / 3hc^3$ (s^{-1}) μ_{ul} is the dipole moment

Critical density: $\mathbf{R_{col}} \sim A_{ul}$ => $\mathbf{n_{crit} = A_{ul} / n\sigma}$; $\mathbf{R_{col} > A_{ul}}$ => thermlized: $\mathbf{R_{col} < A_{ul}}$ => subthermal

Density, number density, column density:

$\boldsymbol{\rho = \mu m_H n}$ [g cm^{-3}] μ = mean molecular weight, **n** = number density of particles [cm^{-3}]

Solar metallicity gas mass fractions: H: **X=0.7**; He: **Y = 0.28**; "metals": **Z = 0.02**. **X+Y+Z = 1**

$\boldsymbol{\mu = [X + Y/4 + Z/15.5]^{-1} = 1.3}$ in neutral atomic gas (HI)

$\boldsymbol{\mu = [X/2 + Y/4 + Z/15.5]^{-1} = 2.37}$ in molecular gas where H is in H_2

$\boldsymbol{\mu = [2X/ + Y/4 + Z/15.5]^{-1} = 0.68}$ in molecular gas where H is in H^+ (HII), Y & Z are neutral

$\boldsymbol{\mu = [2X/ + 3Y/4 + Z/2]^{-1} = 0.62}$ in fully ionized gas (stellar interior)

Interstellar Medium (ISM):

Boltzmann equation: $N_u / N_l = (g_u / g_l) \exp(-[E_u - E_l]/kT)$

Maxwell-Boltzmann velocity distribution function:

$N_v\, dv = n\, (m / 2\pi\, kT)^{3/2} \exp(-mv^2 / 2kT)\, 4\pi v^2\, dv$

Most probably V: $v_{mp} = (2kT/\mu m)^{1/2}$; $v_{rms} = (3kT/\mu m)^{1/2}$; Sound speed: $c_s = (kT/\mu m)^{1/2}$

Saha equation:

$N_{i+1} / N_i = [2kT\, Z_{i+1} / P_e Z_i][2\pi m_e kT/h^2]^{3/2} \exp(-X_i / kT)$ X_i = ionization potential

Z_i = Partition function of ionization stage i. $P_e = n_e kT$ = electron pressure.

Phases of the ISM: Molecular Clouds (H_2), HI clouds, HII regions, hot ISM (HIM)

Contents of the ISM: gas, dust, radiation, cosmic rays, magnetic fields

Jeans criterion for **gravitational collapse**: $V_{esc} > c_s$ => $M_J = (5k\mathbf{T} / G\mu m_H)^{3/2} (3 / 4\pi\rho)^{1/2}$

Column density: $N(H_2) = n(H_2)\, L = M(H_2)$ / [area] (cm^{-2}) : $\rho\, N(H_2) = \mu m_{\_H}\, n(H_2)L$ (g cm^{-2})

Extinction (ISM dust): A_V=1 magnitude ⇔ $N(H_2) \sim 10^{21}$ (cm^{-2}); A_λ is roughly proportional to A_V /λ

HII regions. Photo-ionization balance in uniform density medium:

$Q = L(LyC) = (4/3)\, \pi\, R^3_s\, n^2_e\, \alpha_B$ $\alpha_B = 2.6 \times 10^{-13}\ cm^3\ s^{-1}$

R_s = "Stromgren radius" ~ $(3Q / 4\pi\,\alpha_B)^{1/3}\, n_e^{-2/3}$

External photoionization of a cloud with radius R_o: Flux $F = Q / 4\pi D^2 = (1/3)\, n_e^2\, \alpha_B\, R_o$

Emission measure, $EM = n^2_e\, L$ (cm^{-3} pc) ~ 4.9×10^{17} I(Hα) where I(Hα) is in (erg s^{-1} cm^{-2} $arcsec^{-2}$)

Shocks & Ionization fronts:

Dense (D-type) fronts: Expanding HII regions sweep-up dense shells expanding with shock-speed V_s:

Pressure in the shell, $P_{shell} \sim \rho_o V_s^2 \sim P_{HII} \sim n_e kT_{II} = \rho_{ps}\, c_s^2$:

Post-shock density: $\rho_{ps} \sim \rho_o\, (V_s/c_s)^2 = \rho_o\, \mathcal{M}^2$ where $\mathcal{M}$ is the Mach number.

Blast waves, winds, and expanding HII regions into uniform density media, density = ρ_o**:**

The Sedov "trick": V = dR/dt => R/t Mass of a swept-up dense shell, radius R is $\mathbf{M_s = (4/3)\pi R^3 \rho_o}$

Energy Conserving (E-cons) => Momentum conserving (P-cons) after a cooling time has elapsed.

Blast (E-cons): $E_o = (1/2)M_s V_s^2 \sim R^5/t^2$ => $\mathbf{R \sim (E_o/ \rho_o)^{1/5}\, t^{2/5}}$:(P-cons) :$P_o = M_o V_s \sim R^5/t^2$=> $\mathbf{R \sim (Po/\rho_o)^{1/4} t^{1/4}}$

Wind (E-cons): $1/2[dM_w/dt]V_w^2 = (1/2)M_s V_s^2/t \sim R^5/t^3$ => $\mathbf{R \sim C\, t^{3/5}}$: (P-cons): $\mathbf{R \sim C'\, t^{1/2}}$

HII expansion (E-cons): $\mathbf{R \sim C\, t^{4/7}}$ In all cases, the shock speed is $V_s \sim dR/dt$