Bremsstrahlung is a German word that literally means “braking radiation”. It is the radiation process that occurs when charged particles decelerate (brake) by collisions with other particles.

11.1 THERMAL BREMSSTRAHLUNG

Thermal Bremsstrahlung is the emission from a gas that is at least partially ionized and therefore contains charged particles. It is called thermal bremsstrahlung because it is assumed that the gas is in a thermal equilibrium. More specifically, the gas has the Maxwellian distribution of velocities to be expected from gas in thermal equilibrium.

For (relative) simplicity, in this section we shall look at the case of a fully ionized, pure hydrogen gas. As such, it is composed of electrons and protons in equal numbers. To be fully ionized due to thermal processes, the temperature will be significantly above 10,000K. We further assume that the gas is of sufficiently low density that only two particle collisions occur and that the gas is optically thin, so that a photon, once generated, escapes the system without further interaction.

Three kinds of collisions can occur. Electron-electron and proton-proton collisions are very weak emitters of radiation because there is no dipole component to the collision. Thus the collisions between electrons and protons are the dominant source of radiation.

To derive the integrated output of such a plasma we start with the formula for the emissivity:

$$4\pi j_\nu d\nu = \int \left(n_e f(v) d^3v\right) * v * n_i * E_\nu(v) d\nu \qquad (11.1)$$

where $4\pi j_\nu d\nu$ is the power emitted per unit volume of gas into 4π steradians between frequencies ν and $\nu+d\nu$. n_e is the density of electrons and n_i is the density of ions (protons) which is the same in this case. v is the velocity of a collision and f(v) is the probability distribution of velocities. E_ν(v) is the energy emitted at frequency ν from a collision at velocity v averaged over all impact distances.

The logic of this expression can be understood by inspection. n_e is the number of electrons in a cubic centimeter of gas and the first term in parentheses thus represents the

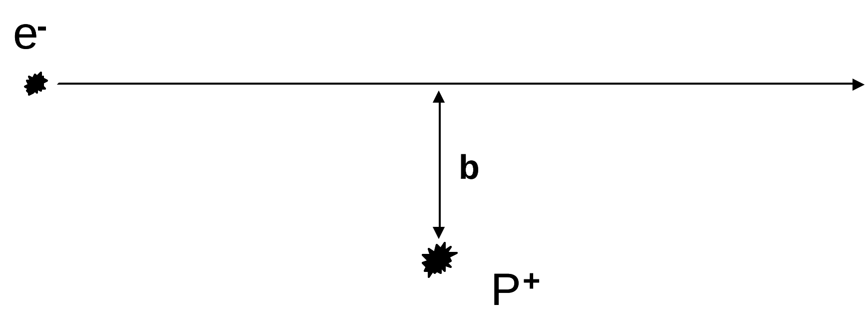

Figure 11.1: The impact parameter, b, is the distance of closest approach that an electron would experience passing by the proton.

We begin by deriving an expression for $E_\nu(v)$. Each collision is characterized by the impact parameter as shown in Figure 11.1. During the collision, the acceleration of the electron is given by

$$a = \frac{e^2}{r^2 m} = \frac{e^2}{b^2 m} \tag{11.2}$$

Where e is the charge of the electron, m is the electron mass, and b is the impact parameter. The impact occurs for a period of time given by

$$t = \frac{b}{v} \tag{11.3}$$

So that the emission frequency is

$$u = \frac{n}{2pb} \tag{11.4}$$

Then, by Larmor's formula, we have

$$E = \int_0^t P(t)dt = \frac{2e^2}{3c^3}\int_0^t a^2 dt \tag{11.5}$$

So we have

$$E = \frac{2e^2}{3c^3} a^2 t = \frac{2e^2}{3c^3}\frac{e^4}{m^2 b^4}\frac{b}{v} \tag{11.6}$$

So we find that

$$E(b) = \frac{2e^6}{3m^2 c^3 b^3 v} \text{ ergs/s} \tag{11.7}$$

are emitted at $\nu = v/2\pi b$ from a collision at v and b.

Since ν and b are inversely related, we write that

$$E(v)du = E(b)2pbdb \tag{11.8}$$

Hence

$$E_u(n) = E(b)2pb\frac{db}{du} \qquad (11.9)$$

But

$$u = \frac{v}{2pb} \quad \text{so} \quad \frac{du}{db} = \frac{-v}{2pb^2} \qquad (11.10)$$

And combining we find

$$E_u(n)du = \frac{8p^2e^6}{3m^2c^3v^2}du \qquad (11.12)$$

Which is the form needed for equation 11.1 The other term needed for equation 11.1 is the distribution of velocities. Since it is assumed this gas is in thermal equilibrium, we have

$$f(v)d^3v = \left(\frac{m}{2pkT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} 4pv^2dv \qquad (11.13)$$

Which it the Maxwellian distribution of velocities.

Combining equations 11.12 and 11.13 into equation 11.1, we find

$$4pj_u = n_e n_i \int_{v_{\min}}^{\infty} \left(\frac{m}{2pkT}\right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} 4pv^3 \frac{8p^2e^6}{3c^3m^2v^2} dv \qquad (11.14)$$

Where $v_{\min}$ is the minimum impact velocity at which a photon of energy ν can be emitted, which is set by

$$\frac{1}{2}v_{\min}{}^2 = hu \qquad (11.15)$$

The integral can be evaluated directly and yields

$$4pj_u = n_e n_i \frac{8pe^6}{3mc^3}\left(\frac{2p}{kTm}\right)^{\frac{1}{2}} e^{-\frac{hu}{kT}} \qquad (11.16)$$

Which, when evaluated, becomes

$$4\pi j_{\nu} = 5.44x10^{-39} \frac{n^2}{T^{\frac{1}{2}}} e^{-\frac{h\nu}{kT}} \text{ erg cm}^{-3} \text{ s}^{-1} \text{ Hz}^{-1} \quad (11.17)$$

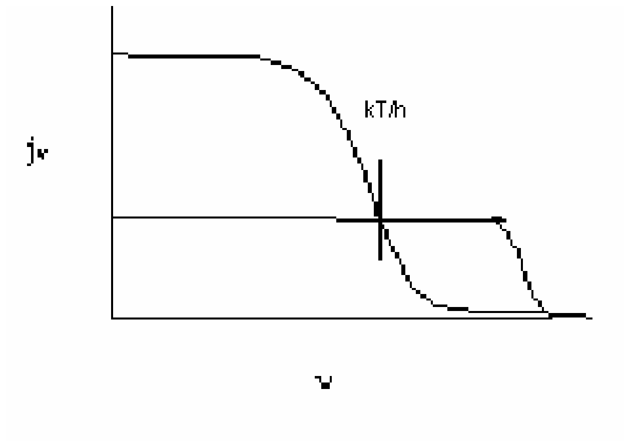

Figure 11.2: The spectrum of a pure hydrogen, fully ionized plasma is essentially flat up until the thermal cutoff near hν=kT.

This result is plotted in Figure 11.2. At low frequency, the spectrum is nearly constant, and becomes an exponential decay as the photon energy approaches the kinetic energy of the average electron. It is this simple function that is fit to the spectra of many x-ray sources.

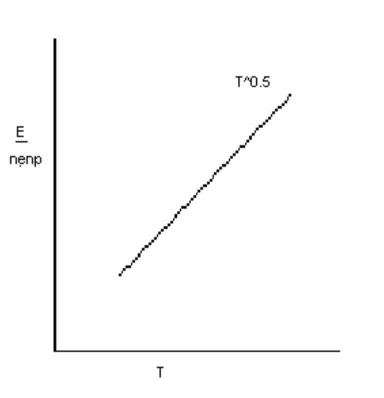

Figure 11.3: The emissivity of a purely ionized gas rises as the square root of the temperature.

Carrying the calculation to its conclusion, we find the total emission per cubic centimeter of plasma is

$$P = 4p \int j_u\, du = 1.4x10^{-27} T^{\frac{1}{2}} n^2 \quad \text{ergs cm}^{-3} \text{ s-1} \qquad (11.18)$$

which shows the power emitted by a cubic centimeter of gas.

Inspection of this equation is instructive as to the nature of bremsstrahlung emission. The output is dependent on only two astrophysical parameters, the density of the gas (in particles per cubic centimeter) and the temperature. The power emitted rises as the square of the density but only as the square root of the temperature. For convenience we bundle all of the physics into a function Λ(T) which, in the case of fully ionized pure hydrogen, is given by

$$\Lambda(T) = 1.4x10^{-27}\sqrt{T} \qquad (11.19)$$

And the power is given by

$$P = n^2 \Lambda(T) \qquad (11.20)$$

EXAMPLE 11.1

An example of a radiating ball.

Of course, the energy source that is drawn upon as the volume of gas radiates is the thermal energy of the particles. After each collision, the electron is moving, on average, slower. Absent the input of new energy, the temperature of the plasma will drop. The electrons lose their energy first, but it can often take considerably longer for the protons to transfer their thermal energy to the electrons, and the two populations of particles can have somewhat different temperatures.

We can see how long it takes for the plasma to cool by considering that the total energy resident in the electrons is given by:

$$E = \frac{3}{2} nkT$$

So that the timescale for significant loss of energy is given by

$$t = \frac{E}{P} = \frac{3nkT}{2x1.4x10^{-27} T^{\frac{1}{2}} n^2} = \frac{T^{\frac{1}{2}}}{n} 1.5x10^{11} \text{ seconds}$$

Or, if the energy in the protons can be transferred, the timescale will be twice as long.

EXAMPLE 11.2

Instability of star-like plasma for x-ray stars.

A remarkable feature of this result is that it can be used to estimate the density of an emitting plasma, see example 11.3.

EXAMPLE 11.3

Flare.

11.2 THERMAL BREMSSTRAHLUNG WITH LINES

As was discussed in Chapter 5, the level of ionization of gas depends heavily on its temperature. There is very little ionization of cosmic composition gas below 10^4K rising to almost total ionization above 10^8K. So the results in the preceding section are correct only in the hottest plasmas. In most environments, there are significant numbers of atoms and ions and they change completely the output of thermal bremsstrahlung.

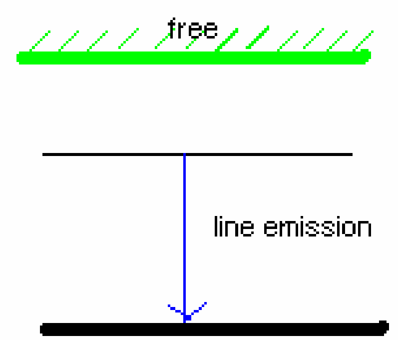

Figure 11.4: When an atom or ion is excited, the electron is raised to a higher energy state that is either a bound or free state. When the electron decays from a bound state it emits an emission line. This process can dominate the gas emission.

When an atom undergoes a collision with an another particle its internal electrons can be excited to higher bound levels or stripped entirely into a free state. These electrons can then fall back toward their base state, emitting photons in the process. From a free state, this is called free-bound emission and from a bound state this is simply line emission.

The atomic physics involved in calculating the response of an atom in a plasma is quite complicated. The quantum mechanics of transition must be calculated for each atomic level in each ionization state of each element in the gas. Nonetheless, astrophysicists have done the work and have provided the results so that can be used to interpret data.

A spectrum from an actual source is usually generated by gas at a range of temperatures showing a range of ionization states. Each ionization state of each element has a different set of lines related to a temperature range. So, each line can be used as a diagnostic as the output of that line is related to the density of electrons times the density of ions just as in equation 11.14. Thus the ability to detect and measure intensities of emission lines is a rich diagnostic of the state of the emitting matter.

At any given temperature, a gas of cosmic composition, given enough time will reach ionization equilibrium, and the distribution of ions becomes predictable. The total output of each species can be predicted and the species summed to predict the total output. Because of the inherent complexity, the emission physics is all bundled into the Λ(T) emissivity function introduced in equation 11.20. So we once again have

$$P = n^2 \Lambda(T)$$

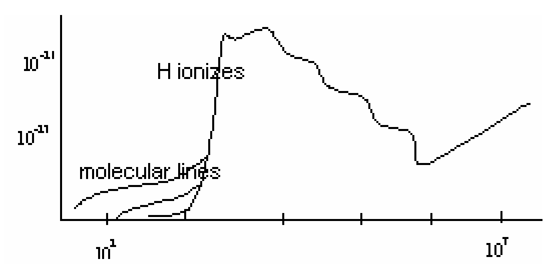

Figure 11.5: The emissivity of a cosmic composition gas is plotted as a function of temperature.

This function has theBut this time, there is a great deal of complexity in the form of Λ(T). This information is provided in tabular or graphical form to the astrophysicist who wishes to interpret data. The power varies as the square of the electron density in most cases.

Figure 11.5 gives a summary of Λ(t) over a broad range of temperatures. At low temperatures, where the kinetic energies of motion are too low to excite many atomic transitions, the rotational states of molecules can provide some emission lines and cooling.

At about 10,000K hydrogen starts to ionize and there is a dramatic increase in the emissivity, most of it coming from the Lyman α line. As the temperature continues to rise, the fraction of hydrogen that remains bound starts to drop and the emission lines become weaker. However, helium has a higher ionization potential and still has ions with bound electrons, so that the emission lines of HeI and HeII dominate the output of the plasma. At higher temperatures still, other heavier elements become dominant and a thicket of bright spectral lines emerges. As the temperature continues to rise successively heavier elements become fully ionized and the total emission keeps dropping. By 10^7K, only high ionization states of iron remain in significant quantities. So, above 10^7K, the fully ionized approximation becomes valid and the emissivity starts to rise again, this time as the square root of temperature.

11.3 NON-THERMAL BREMSSTRAHLUNG

This section to be written later. Will explain thick target bremsstrahlung and other variations on the theme.

EXERCISES