CHAPTER 13: PHOTON SCATTERING

When a photon already has been created and is traveling through a gas, a number of interactions can affect its progress. In this chapter we address what happens when photons interact with electrons in simple two-body collisions. The net result is a scattering of the photon. Simple absorption is not allowed because, viewed in the center of mass frame, this would constitute a change in the rest mass of the electron.

13.1 COMPTON SCATTERING

Compton scattering is one of the main forms of interaction between x-rays and matter. It is the primary source of opacity of medical x-rays passing through the human body, but it does not destroy the photon. Compton scattering changes the energy of the photon and redirects it.

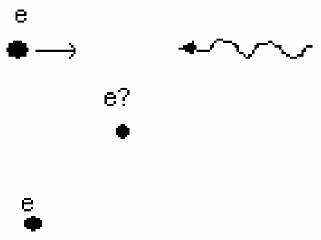

Figure 13.1:

Consider Figure 13.1. At the top an electron and a photon approach each other in the zero momentum frame of reference. The photon has momentum E/c and the electron mv in the opposite direction. One can view the interaction as a temporary absorption of the photon. So, in the second panel, we have a particle that resembles an electron but has a rest mass of

$$mc^2 = m_e c^2 + \frac{2E}{c}$$

Although physics does not yet understand why this is forbidden, the laws of quantum mechanics only allow a very specific mass for an electron, so this temporary particle sheds the extra mass/energy in the form of a new photon as in the third panel of Figure 13.1. The new photon will have the same energy as the incident photon (in the center of mass frame) but it will have a new direction.

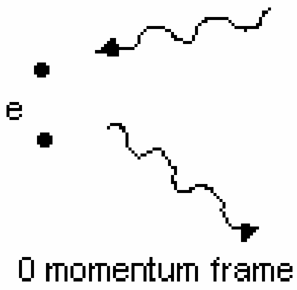

Figure 13.2:

The angle through which the photon is scattered in any given interaction cannot be predicted like it can when two billiard balls collide. So, we analyze the effect in terms of the initial and final outcomes, using conservation of momentum and energy in the usual way.

An electron has momentum p=mv and a photon has momentum p=h/λ. We work in the plane of the scattering and in the frame in which the electron is initially stationary. We consider the two particles to be approaching each other along the x-axis, so that conservation of energy gives us

$$\frac{hc}{\lambda_i} = \frac{hc}{\lambda} + \frac{1}{2}mv^2$$

Where λ_i is the wavelength of the photon initially, and λ is the wavelength after scattering. V is the velocity imparted to the electron from the photon.

Conservation of momentum in the x direction (that of the initial photon) gives

$$\frac{h}{\lambda}\cos\theta + mv_x = \frac{h}{\lambda_i}$$

And in the y direction:

$$\frac{h}{\lambda}\sin\theta + mv_y = 0$$

Squaring both sides of the momentum equations, summing and comparing to the energy equation we find

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda_i} - \frac{hc}{\lambda} = \frac{1}{2m}\left[\left(\frac{h}{\lambda}\sin\theta\right)^2 + \left(\frac{h}{\lambda_i} - \frac{h}{\lambda}\cos\theta\right)^2\right]$$

Which gives us

$$\lambda - \lambda_i = \frac{h}{2mc}\left[\frac{\lambda_i}{\lambda} + \frac{\lambda}{\lambda_i} - 2\cos\theta\right]$$

In the case where the photon energy is substantially below the rest energy of the electron, the wavelength will not change significantly in the scattering, so

$$\lambda - \lambda_i = \frac{h}{mc}(1 - \cos\theta)$$

Where

$$\lambda_c = \frac{h}{mc} = .024\text{Å}$$

Is known as the Compton wavelength. What this equation tells us is that when a photon scatters off an electron, its wavelength is increased by up to twice the Compton wavelength. If the photon initially has a wavelength much greater than this, then the the shift is quite small. However, if the initial photon is an x-ray or gamma-ray, then the shift in wavelength and loss of energy can be quite substantial.

In everyday life, we encounter Compton scattering most prominently in x-ray imaging of the human body. To a medical x-ray (λ_i~0.2Å) the electrons in the oxygen (and other low mass elements of the human body) are loosely bound and behave like free electrons. Compton scattering is the predominant form of x-ray interaction in soft tissue, and can fog x-ray plates if not properly removed.

13.2 INVERSE COMPTON SCATTERING

An interesting variation on scattering occurs when the electron is relativistic. It can actually impart large amounts of energy into a photon, shifting its wavelength across the electromagnetic spectrum.

Consider the case of an electron with energy $\gamma m_0 c^2$ scattering off a photon of frequency ν. If one transfers to the rest frame of the electron the electron will have zero kinetic energy and the photon will have a frequency $\gamma\nu$. If the photon backscatters off the electron it can essentially reverse direction without changing frequency. However, when one converts back into the original observer's frame of reference, the photon gains another factor of γ in frequency. Thus the net effect to upshift the frequency by as much as a factor of γ2. We have

$$n = g^2 n_i$$

Of course, not all photons will experience the full shift of γ^2 in frequency, depending on scattering angle. The effect is to scatter photons across a broad range of the spectrum up to a factor of γ^2 higher than its original state.

Because, in astrophysics, relativistic electrons often have a power law distributions of energy and therefore of γ, it is natural for the spectrum of the scattered photons to exhibit a power law distribution as well, even if they were thermally distributed originally. Because of the need for many electrons to scatter the photons, it is rare for a spectrum to be dominated by inverse Compton scattering.