Up until now, we have assumed that, once a photon is created or changed, it is free to travel to the observer without further interaction. But in many cases the photon simply cannot do that. If the photon is deep inside a dark object, it cannot travel very far at all, and must be reabsorbed. The photons must come to some kind of equilibrium where they are absorbed and re-emitted at each wavelength in equal numbers. Only when they are emitted from very near the surface can they escape, carrying with them the equilibrium spectrum of this blackbody.

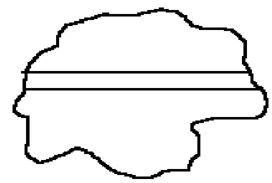

Figure 14.1: An optically thick object has photons attempting mover through it along some path as shown schematically.

14.1 RADIATIVE TRANSFER

Until now:

$$I_\upsilon = \int j_\upsilon dx \tag{2.1}$$

but what if absorption occurs?

$$dI_\nu = j_\upsilon - \kappa_\upsilon I_\upsilon \tag{2.2}$$

But what if it gets very thick?
Then dI_υ=0

$$I_\upsilon = \frac{j_\upsilon}{\kappa_\upsilon}$$

Let us look at this more closely
Consider the patricles doing the absorbing and emitting.
Take two states, upper u and lower L.
Separate them by E_U-E_L=hν. Then Boltzman tells us:

$$\frac{N_L}{N_U} = \frac{g_L}{g_U} e^{\frac{h\nu}{kT}}$$

Also, in equilibrium

$$N_U[A_{UL} + I_\nu B_{UL}] = N_L B_{LU} I_\nu$$

A_{UL} is the term for spontaneous emission
B_{UL} is the term for stimulated emission
B_{LU} is the term for absorption

Combine to find

$$e^{\frac{h\nu}{kT}} B_{LU} I_\nu = \frac{g_U}{g_L}[A_{UL} + I_\nu B_{UL}]$$

Let T go to ∞ so I goes to ∞

Then

$$B_{LU} = \frac{g_U}{g_L} B_{UL}$$

So, in general and in equilibrium

$$e^{\frac{h\nu}{kT}} B_{UL} I_\nu \frac{g_U}{g_L} - B_{UL} I_\nu \frac{g_U}{g_L} = \frac{g_U}{g_L} A_{UL}$$

So $I_\nu = \frac{A_{UL}}{B_{UL}} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$

Then, it can be shown using quantum mechanics and box normalization that

$\frac{A_{UL}}{B_{UL}} = \frac{2h\nu^3}{c^2}$

So $I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)}$

This is the famous Planck formulation for blackbody radiation, and it has units (cgs) of ergs/cm^2/s/st/Hz.

Next we convert from frequency units

$I_\nu d\nu = \frac{2h\nu^3}{c^2} \frac{1}{\left(e^{\frac{h\nu}{kT}} - 1\right)} d\nu$

And $\nu = \frac{\lambda}{c}$ so $d\nu = \frac{c}{\lambda^2}$

$I_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$

Where the units are now ergs/cm^2/s/st/cm. Note that the second cm term is per centimeter of wavelength and thus does not multiply against the cm^2 of emitting area.

It is interesting to find the peak of this curve by differentiating and setting it to zero. We find

$\frac{5\lambda kT}{hc} e^{\frac{hc}{\lambda kT}} - 1 = e^{\frac{hc}{\lambda kT}}$

Letting λT=x we see that the above is of the form

$$cx - 1 = e^{\frac{\alpha}{x}}$$

Which only has one solution for x, leading us to the conclusion that

$\lambda_{max} = \frac{2.9x10^7}{T}$ where T is in Kelvin and λ is in Angstroms.

EXAMPLE

The Sun is everyone's favorite blackbody. We know its spectrum peaks at 5500Å and that it has a radius of $7x10^{10}$cm.

From the Wein law we find that

$$T = \frac{2.9x10^7}{5500} = 5500K$$

And then

$L = 5.67x10^{-5}\pi(7x10^{10})^2(5500)^4 = 4x10^{33}$ ergs/s

Next we look at the low frequency end of the spectrum, where hν<<kT.

$I_\nu d\nu = \frac{2h\nu^3}{c^2}\frac{1}{\left((1+\frac{h\nu}{kT}-1\right)}d\nu$

$I_\nu = \frac{2\nu^2}{c^2}kT$

Which is known as the Rayleigh-Jeans Law

This was the classical form of blackbody radiation, before Planck had the idea of quantizing the radiation. Because the intensity continues to rise to higher frequencies, the total power radiated is infinite. This was known as the "Ultraviolet Catastrophe".

The Rayleigh-Jeans law is often used to characterize radio sources where ν is so low that it must be below kT in an astrophysical setting. Of course, we are detecting nonthermal (usually synchrotron) radiation, but astronomers use this "Temperature Brightness" to characterize the surface brightness of the source. If the emitting area is known, then a temperature can be calculated. Such Temperatures are often in the gigaKelvins.

Finally we look at the total power radiated by a blackbody.

$$I = \int I_\nu d\nu = \frac{2}{h^2c^2}\int \frac{(h\nu)^3}{e^{\frac{h\nu}{kT}} - 1} d\nu$$

$$I = \frac{2}{h^2c^2}(kT)^3 \frac{kT}{h}\int_0^\infty \frac{x^3}{e^x - 1} dx$$

Where, using complex calculus one can find that the definite integral has the value of $\pi^4/15$.

So $I = \frac{\sigma}{\pi}T^4$ where $\sigma = \frac{2\pi^5k^4}{15h^3c^2} = 5.67x10^{-5}$ in cgs units.

From a flat surface of area A we have

$$P = A\int_0^{2\pi}\int_0^{\pi/2} I\cos\theta(\cos\theta\, d\theta d\varphi) = \pi I$$

And so

$$P = \sigma A T^4$$

Which is known as the Stefan-Boltzman Law and has units of ergs/cm^2/s in cgs.

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EXERCISES

1. Neutral hydrogen emits radio waves with a wavelength of 21cm. What is the frequency of this radiation?
2. What is the wavelength (in Å) of a 1000eV (1keV) x-ray?

EXAMPLE