There are two systems of units that are in general use within the sciences, MKS and CGS. The MKS system (which uses meters, kilograms and seconds) tends to be favored by physicists. For reasons unknown, however, the CGS (centimeters, grams, seconds) units are heavily favored by astronomers and astrophysicists. CGS units do have a certain convenience in electromagnetic theory. The use of Gauss for the measure of magnetic field, leads to the elimination of the μ_0 and ε_0 in calculations. As we are discussing astrophysics, we use the CGS conventions throughout.

2.1 FREQUENCY AND WAVELENGTH

Photons, the quantized packets of electromagnetic radiation that carry information through space, all travel at the speed of light in vacuum. Each photon has direction, frequency, wavelength, and polarization. Furthermore, the distribution of these properties in a set of emitted photons carries information. We need units for describing the radiation.

Since all photons travel at the speed of light (3×10^{10} cm/s), we can write the relationship:

$$\lambda \nu = c \tag{2.1}$$

EXAMPLE

What is the frequency and energy of a Hydrogen Lyman α photon (1216Å)?
Answer:
1216Å = 1.216×10^{-5} cm
$\nu = c/\lambda = 3 \times 10^{10}/1.216 \times 10^{-5} = 2.5 \times 10^{15}$ Hz
$\varepsilon = h\nu = 6.6 \times 10^{-27} \times 2.5 \times 10^{15} = 1.65 \times 10^{-11}$ ergs

---

where λ is the wavelength, ν is the frequency, and c is the speed of light. This simply states that the distance a wave travels during one oscillation (the wavelength) times the number of oscillations per second will equal the distance traveled in one second. But, since the speed of light is a fixed parameter of the universe, we have an immutable relationship between wavelength and frequency. We can refer to them interchangeably.

Another basic property of the photon is its energy, given by the formula:
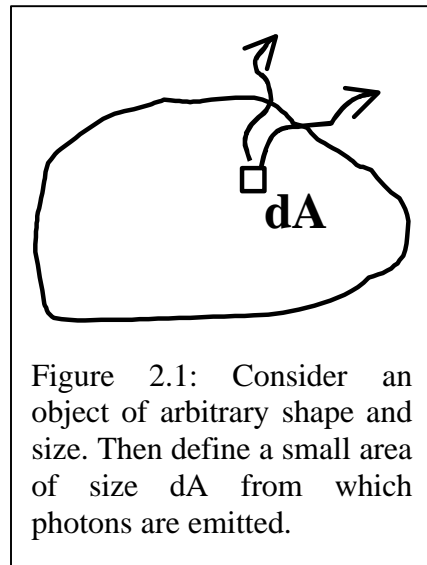
$$\varepsilon = h\nu \tag{2.2}$$

Figure 2.1: Consider an object of arbitrary shape and size. Then define a small area of size dA from which photons are emitted.

where ε is the energy of the quantum in ergs, ν is the frequency in oscillations per second, and h is Planck's constant (6.6×10^{-27} erg seconds). Because of the simple relationship, it is equally valid to refer to a photon by its energy as well as its frequency or wavelength. Astrophysicists use all three of these of options in different situations.

The basic unit used for frequency is the Hertz (written Hz) which is one oscillation per second. This is the only unit used for frequency, although KHz is used to represent a thousand oscillations per second, MHz is short for a million oscillations per second and GHz is short for 10^9 per second.

When referring to the energy of a photon, astrophysicists usually use the electron volt (eV). This is the amount of energy released by moving one electron through a one Volt potential, and has the value of 1.6×10^{-12} ergs. A 1.0eV photon has a frequency of 2.42×10^{14}Hz, falling in the near infrared. Energy units are the favorite of high energy astrophysicists, who mostly deal with energy properties of photons as opposed to wave properties. In the x-ray, the keV (kiloVolt) is favored, and at higher energies yet, gamma ray astronomers use MeV.

One variation on energy units is the "inverse centimeter" (cm^{-1}) sometimes used by infrared and microwave astronomers. One simply takes the reciprocal of the wavelength of the photon (in centimeters) and uses that number. Since it scales as the inverse of the wavelength it is proportional to
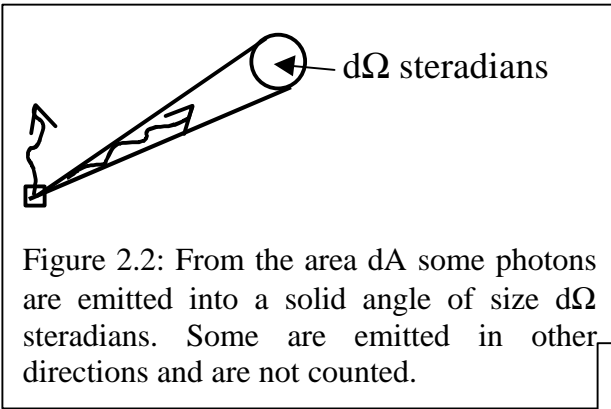

Figure 2.2: From the area dA some photons are emitted into a solid angle of size dΩ steradians. Some are emitted in other directions and are not counted.

frequency and energy.

The most common unit used for wavelength is the Angstrom (Å) which is defined as 10^{-8}cm. Yellow light has a wavelength of about 5500Å. From the near infrared into the x-ray, the Angstrom is the favored unit. However, in the far infrared to the radio the wavelengths get very large when quoted in Å, so longer units like microns, millimeters and centimeters are employed. The centimeter is the preferred wavelength unit for the radio band.

2.2 INTENSITY UNITS

While we can refer to individual photons using the units above, much more information is to be found in the group properties of the radiation. How many photons are emitted, and how many are generated as a function of frequency? This
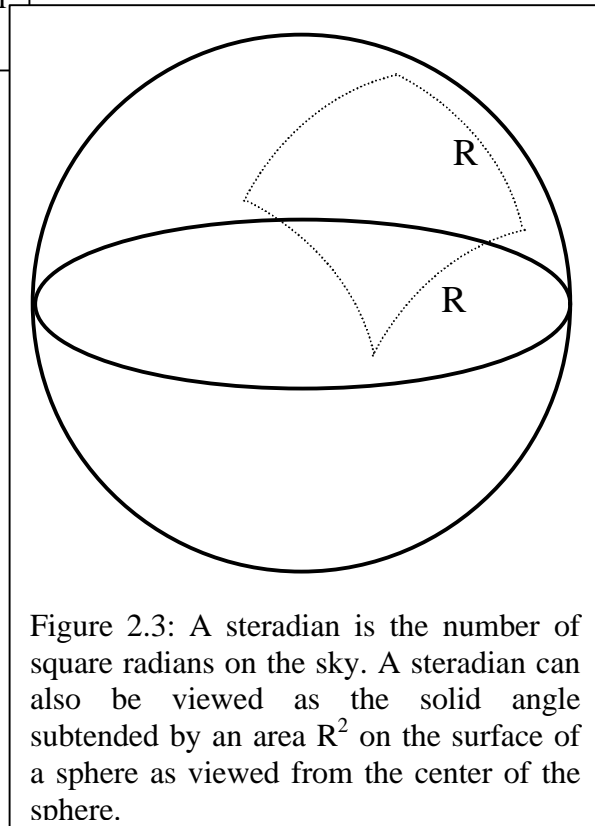

Figure 2.3: A steradian is the number of square radians on the sky. A steradian can also be viewed as the solid angle subtended by an area R^2 on the surface of a sphere as viewed from the center of the sphere.

is the root of spectroscopy, the science that allows us to analyze radiation and infer the physical state of the emitting source.

We need units that will allow us to work with the quantity and distribution of radiation from an arbitrary source. In Figure 2.1 we show an arbitrary object and define a small region of size dA on its surface. From that region we shall measure the radiation emitted. We can count the total number of photons emitted as a function of frequency and have much of the information needed. However, there is no guarantee that it will emit uniformly in all directions.

Figure 2.2 shows how we take into account emission as a function of angle. We define a small solid angle $d\Omega$ and then count the number of photons (n) radiated from dA into $d\Omega$. The units of $d\Omega$ are steradians (see figure 2.3).

So, counting photons, we see n photons per area dA, between frequency ν and $\nu+\delta\nu$, radiating into $d\Omega$ steradians per second. This unit is called *photon intensity* can be written:

$$P(\boldsymbol{n}) = \frac{dn}{dA \cdot d\boldsymbol{n} \cdot d\Omega \cdot dt} = n \quad \text{ph cm}^{-2} \text{ s}^{-1} \text{ st}^{-1} \text{ Hz}^{-1} \tag{2.3}$$

We can convert to a unit that measures energy flux instead of photon flux. Called *specific intensity*, we find the unit by multiplying by $h\nu$, the energy carried by each photon. Thus:

$$I(\boldsymbol{n}) = \frac{de}{dA \cdot d\boldsymbol{n} \cdot d\Omega \cdot dt} = i \quad \text{ergs cm}^{-2} \text{ s}^{-1} \text{ st}^{-1} \text{ Hz}^{-1} \tag{2.4}$$

The next unit is found by removing the solid angle dependence. By integrating over all angles into which photons are emitted, we find the *flux density* of the source. This is given by:

$$F(\boldsymbol{n}) = \frac{de}{dA \cdot d\boldsymbol{n} \cdot dt} = f \quad \text{ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \tag{2.5}$$

Flux density is a basic unit frequently used to describe the intensity of a source in the sky. Since it does not have solid angle dependence, it is an appropriate unit to use when viewing a star-like object which cannot be resolved in angle. In fact, flux density is used so commonly that it has been given its own unit called the Jansky, in honor of the inventor of the radio telescope.

$$1\text{Jy} = 10^{-23} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \tag{2.6}$$

Because the Jy is a unit of flux density, it is often presented as a spectrum, in which the value of F is plotted as a function of ν. However, we often wish to know the total emitted by the source across the entire spectrum. In that case we use flux, which is defined by:

$$F = \int f(\mathbf{n})d\mathbf{n} = f \ \ \text{ergs cm}^{-2} \ \text{s}^{-1} \tag{2.7}$$

Flux can be understood as simply the amount of radiant power passing through a given square centimeter of source or detector. It is flux we feel warming our bodies on the beach. It is flux that our eyes detect as the brightness of a star.

Brightness, as used by the astrophysicist, has a very specific meaning. It is used to measure the surface brightness of a resolved object like a nebula or galaxy. Returning to specific intensity (equation 2.4) and this time integrating over frequency instead of solid angle, we find the brightness:

$$B = \int I(\mathbf{n})d\mathbf{n} = b \ \ \text{ergs cm}^{-2} \ \text{st}^{-1} \ \text{s}^{-1} \tag{2.8}$$

One remarkable aspect of brightness is that it is conserved. While the flux from a source may dim as it recedes, the brightness does not. Within the solid angle subtended by the source, the brightness remains the same. This effect is relatively easy to understand when one considers the brightness of a uniform, infinite wall. It appears exactly the same no matter how far we are from it. The fall in flux is compensated by the greater area per angle.

2.3 MAGNITUDES

By far the most common unit used to describe the flux from stars is the magnitude scale. Originally derived from a brightness classification scale created by Ptolemy in the Middle Ages, it is not a simple physical unit like a Jansky. However, it has survived through the centuries as it provides a certain convenience to the visible light astronomer.

Magnitudes are a logarithmic scale. Each magnitude is exactly $\sqrt[5]{100} = 2.51$ times brighter than the next. A star five magnitudes brighter is exactly 100 time brighter. Thus, a star (like α Cen) with m=0 is 100 times brighter than a fifth magnitude star, and 10,000 times brighter than an m=10 star. Magnitudes are also unusual in that, the fainter the object, the larger the value of its magnitude.

Because of this inconvenience, we do not use magnitudes heavily in astrophysics. In this book they are used as a shorthand notation for discussing the brightness of a star, but the physical analysis is completed in more physical units. Nonetheless, because of their prevalence, we discuss their use.

Some Magnitudes

	m	M
Sun	-26.5	4.5
Moon	-13	20
Sirius	-1.5	1.
α Cen	0.	4.5
Antares	1.	

Consider a standard star of magnitude zero, and give it the brightness designation of I_0. Then take a star of intensity I. The magnitude of the second star is given by:

$$m = \log_{\sqrt[5]{100}}(I_0 / I) = 2.5\log(I_0 / I)$$

The Sun, by far the highest intensity source in the sky, has an apparent magnitude of –26.5, the full moon is –13. The brightest night star in the sky is Sirius, with m=-1.5, and the faintest star visible to the naked eye is around m=6. The Hubble Space Telescope can detect stars as faint as m=30.

The magnitudes quoted are *apparent magnitudes*, in that they contain the effects of distance. As a star moves further from Earth, its apparent magnitude value increases as its flux drops. To discuss the intrinsic output of a star, independent of its distance, the *absolute magnitude* was invented. The idea is to pretend that all objects are at the same, standard distance. We measure the apparent magnitude and the actual distance. We then calculate what the apparent magnitude would be if the star were moved to a distance of exactly 10 parsecs. (Note: a parsec is 3×10^{18} cm). We can calculate the change in magnitude.

Take a star of magnitude m at a distance d parsecs. Let us call its intensity as viewed from distance d pc I_d. Let us also say that the intensity as viewed from 10pc is I_{10} and its magnitude is M. We can then write:

$$\frac{I_d}{I_{10}} = \left(\frac{d}{10}\right)^2 \tag{2.9}$$

but from the definition of magnitude

$$m = -2.5\log_{10}\left(I_d / I_0\right) \tag{2.10}$$

and

$$M = -2.5\log_{10}\left(I_{10} / I_0\right) \tag{2.11}$$

so

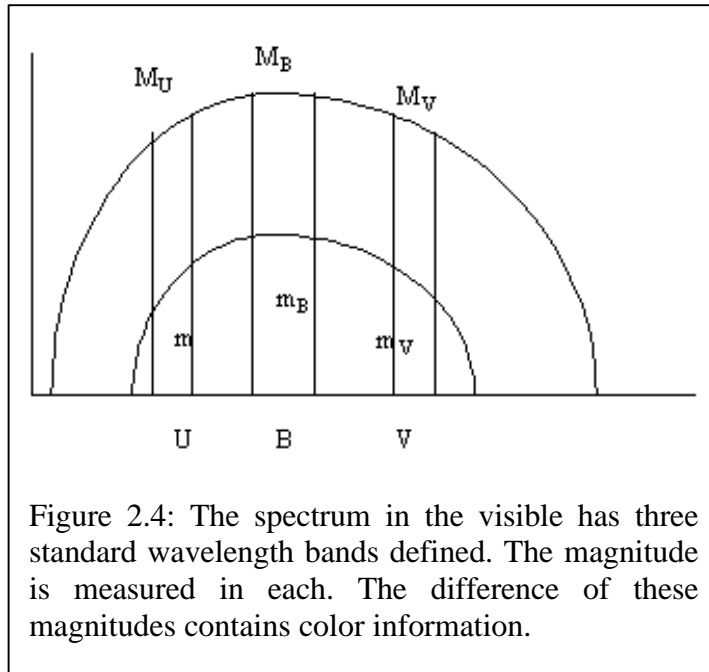$$M - m = -2.5\log_{10}\left(I_{10} / I_0\right) + 2.5\log_{10}\left(I_d / I_0\right) \tag{2.12}$$

or

$$M = m - 2.5\log_{10}\left(I_{10} / I_d\right) = m - 2.5\log\left(\left(\frac{d}{10}\right)^2\right) \tag{2.13}$$

which simplifies to:

$$M = m - 5\log d + 5 \tag{2.14}$$

Thus, if one knows the distance to the star, its apparent magnitude can be directly converted to absolute magnitude. However, without distance information, the absolute magnitude remains unknown.

Figure 2.4: The spectrum in the visible has three standard wavelength bands defined. The magnitude is measured in each. The difference of these magnitudes contains color information.

The magnitude scale was developed during a time when the only band in which stars were observed was the visible. To the extent that the visible band is narrow, the magnitude scale worked well. However, the intensity of objects in the universe can be a strong function of wavelength. Thus the magnitude of an object will vary as a function of the wavelength at which it is being observed. Even across the narrow visible band, the effects of color are significant. Antares is bright in the red, but faint in the blue.

To maintain the magnitude system in the visible, astronomers started referring to magnitudes in certain pre-defined bands. These bands, which are in turn defined by a set of standard filters for the telescope, are called U,B, and V, for ultraviolet, blue and visible, and the magnitudes within the bands are called m_U, m_B, and m_V. If one quotes the magnitude in each of the bands, a sense of the color of the star can be retained. Often, the difference in magnitude between the colors is quoted. For example m_B-m_V is known as a color index. This system does not extend well outside the visible part of the spectrum, and is in limited use in the broad range of astrophysical problems.

2.4 DISTANCE

For the most part, we use the centimeter as the measure of distance. However, we often want a shorthand for large, interplanetary and interstellar distances.

The Astronomical Unit (AU) is used to discuss solar system scales. The definition of the AU is the distance from the Earth to the Sun, and has the value of 1.5×10^{13}cm. For example, Saturn is 10AU from the Sun. But, for interstellar distances, the AU is far too small. The nearest star is almost 300,000AU away.

Two units are in use for interstellar distances, the light year and the parsec. The light year is rarely used in astrophysics, and it is defined as the distance light travels in one year. Its value is 9×10^{17}cm.

The parsec is common use, and is not much larger than the light year, amounting to 3×10^{18}cm. The parsec is defined as the distance at which one AU subtends an angle of one arcsecond. Since there are 206,265 arcseconds in a radian, the parsec is defined as 206,265AU. The closest star sits at 1.3pc, a convenient unit for describing the solar neighborhood.

As move to distances comparable to the size of the Milky Way, the kiloparsec (Kpc) is used. The closest external galaxies are millions of parsecs away, so units of megaparsecs (Mpc) are employed in extragalactic astronomy.

EXERCISES

1. Neutral hydrogen emits radio waves with a wavelength of 21cm. What is the frequency of this radiation?
2. What is the wavelength (in Å) of a 1000eV (1keV) x-ray?
3. An opaque spherical body of radius 1km has a uniform photon intensity of 1 ph/cm^2/s/Hz/st between 10^{14} and 10^{15}Hz. At all other frequencies it does not emit. What is its luminosity in ergs/sec?
4. An astronomer using a telescope discovers that a 12th magnitude star is actually a double binary consisting of four identical stars. What is the magnitude of each individual star?