Astrophysics is the discipline of applying physics to understanding the constituents of the universe. Since nearly all of our knowledge of the universe has been carried to us by electromagnetic radiation, we must first understand the mechanisms that create photons before we can test models of the nature of the objects that emitted them.

Our understanding of light derives first from classical electromagnetic theory as derived in the 19th Century. In the early to mid 20th Century this theory was integrated into quantum mechanics so that we now have a reliable understanding of the physics of photon emission.

In astrophysics we are looking complicated ensembles of particles. The physical status of the material doing the emitting gives rise to the intensity and spectrum of the object, but deep inside, on a microscopic scale, it is the acceleration and deceleration of charged particles that create all the photons.

In this chapter we present a simplified version of the radiation from a single charged particle. In the subsequent chapters we use these simple formulae to explain the output of full systems under certain circumstances.

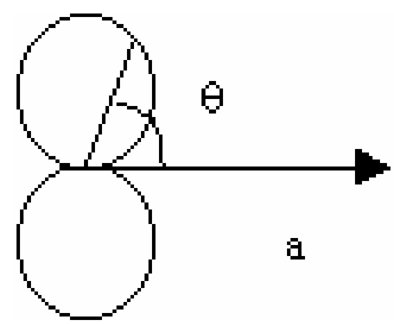

Figure 8.1: The radiation from a particle emits in a doughnut-shaped pattern. The radiation is primarily perpendicular to the direction of acceleration.

2.1 AN ISOLATED CHARGE

We start with the simplest case, which is that of an electron at rest. Classical E&M tells us that the power radiated per steradian is given by:

$$\frac{dP}{d\Omega} = \frac{e^2 a^2}{4\pi c^3} \sin^2 \theta \qquad (8.1)$$

Where θ is the angle of the radiation relative to the direction of the acceleration, *a*, of the particle. It does not matter what force causes the acceleration, the output is only dependent on the magnitude and direction of the acceleration.

In equation 8.1 *e* is the electric charge of the electron in electrostatic units (esu). We usually are concerned with the output of individual electrons. Protons can emit too, and have the same output as an electron at the same acceleration, yet we normally ignore their output. Because a proton has nearly two thousand times the mass of the electron, its acceleration is typically two thousand times lower and its output 4 million times lower than an electron. Since electrons and protons are usually present in similar numbers, any proton signal is usually unobservable and of no significant impact on the system.

Equation 8.1 also assumes that the charge is exactly that of the electron. However, larger charges can be assembled out of smaller units. If they are accelerated together, as a unit, then the power radiated rises as the square of the number of charges. If the individual charges do not stay coherent, then the power rises linearly with the number of charges. For most cases in astrophysics the particles react individually, and the output is simply the sum of the individuals.

The fact that the radiation goes outward in a ring, perpendicular to the direction of accperation explains why radio antennae on cars are pointed upward. A radio station wishes its signal to blanket the countryside where the listener are and does not wish to communicate with targets directly overhead, so their antennae are pointed skyward and the electrons are acclerated in a vertical direction. The radio waves then spread outward along the ground in all directions. Radio reception is best when the electric field of the wave accelerates electrons along the long direction of the antenna. So car radios are also pointed skyward. It is odd, but true, that should an antenna be pointed directly at you, you would be located at the $\sin\theta=0$ position and unable to see any signal.

We are also interested in the total output of the radiation. By integrating the output over all directions we find:

$$P = \int \frac{dP}{d\Omega} d\Omega = \frac{2e^2 a^2}{3c^3} \tag{8.2}$$

which is known as the Larmor Formula. The total power emitted by the charge rises as the square of its acceleration. This simple formula will be of great use in understanding astrophysical emission mechanisms.

EXAMPLE

An isolated electron is dropped just above the surface of the Earth. How much radiation does it emit while in free-fall? If it is dropped from a height of 10,000cm and is stopped in the first 100microns of rock that it encounters, how much radiation does it emit?

Answer: xxx

2.2 A RELATIVISTIC CHARGE

Astrophysics is blessed with many high energy objects that have particles moving very close to the speed of light. When a relativistic particle is accelerated it still emits, but the situation gets more complicated. Because the direction of the relativistic motion and the direction of the acceleration are not necessarily the same, the symmetry of radiation can be broken.

In the case where the acceleration and velocity are co-linear, the output of the charge is given by:

$$\frac{dP}{d\Omega} = \frac{e^2 a^2 \sin^2\theta}{4\pi c^3 (1-\beta\cos\theta)^5} \tag{8.3}$$

Where β is v/c in the usual way. Since the new term in the denominator is always less than one, the total output in the relativistic case is larger. As β approaches one, the amplification can be quite large. In Figure 8.2 we show the effect of relativity on the doughnut of emission. When β is very close to one, the emission appears to be beamed directly forward, and very little escapes to the sides.

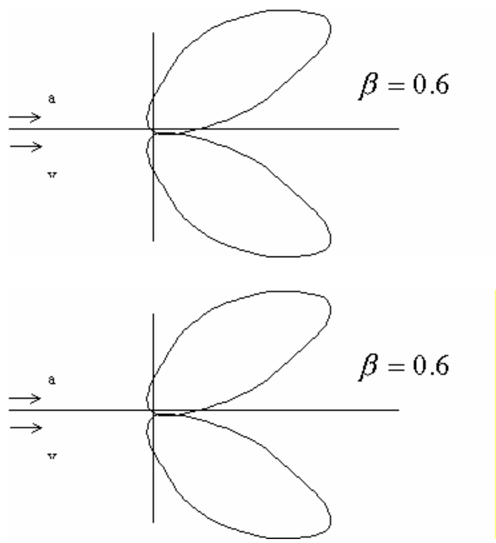

Figure 8.2: When the acceleration and velocity are in the same direction, the doughnut of emission is warped forward. Above, at 60% the speed of light, the region of highest emission is bent forward about 45 degrees. Below, at 95% of c, the radiation appears in a needle, aimed almost straight forward.

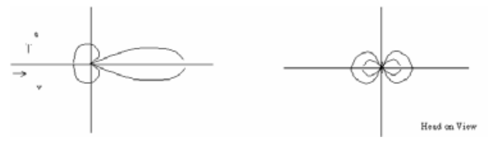

Figure 8.3: When the acceleration and velocity are perpendicular, the doughnut of emission is still warped forward. To the left is shown the radiation pattern as viewed from the side with the velocity to the right and the acceleration upward. To the right is the view of the pattern as seen if the particle were headed directly toward you.

In the case where the acceleration is perpendicular to the direction of motion, the formula is even more complex.

$$\frac{dP}{d\Omega} = \frac{e^2 a^2}{4\pi c^3} \frac{(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi}{(1-\beta\cos\theta)^5} \qquad (8.4)$$

This even more complex formula is dependent on the same θ as before, but also on the angle φ which is the angle with respect to the direction of motion. An example of the pattern is shown in Figure 8.3.

In astronomical settings we rarely see an emitting particle with such precision. There is usually a range of angles involved which washes out the fine structure. However, in some cases, like synchrotron radiation (chapter 10) the forward beaming of particles in a magnetic field can lead to major changes in appearance as a function of angle.

As before, we can integrate over all directions to find the total power radiated. The result is quite similar to the Larmor formula with the addition of a relativistic term.

$$P = \frac{2e^2 a^2}{3c^3}\gamma^2 \qquad (8.5)$$

In the case of co-linear velocity and acceleration, γ is the usual relativistic dilation term given by

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad (8.6)$$

When the acceleration is perpendicular to the velocity, the relativistic effect becomes even stronger

$$P = \frac{2e^2 a^2}{3c^3}\gamma^4 \qquad (8.7)$$

It is noteworthy that the radiation losses rise as the square of γ in the first case and as the fourth power in the second. Thus synchrotrons, wherein the particles are accelerated in circular orbits will lose energy through radiation at a higher rate than in linear accelerators.

2.3 FREQUENCY OF RADIATION

Larmo's formula tells us how much power is radiated, and into which direction it will go, but does not tell us at what frequency to expect the radiation. Since the wavelength of emission is crucial to understanding the environment, we need a simple way to predict where the light is emitted.

The power radiated can be thought of as the sum of waves over time. As is usual in optics, the power is proportional to the square of the sum of the amplitudes driven by an acceleration. A simple explanation can be found in the formula:

$$P(\nu) \propto \left| \int_0^{\tau} a(t) e^{i(2\pi\nu)t} dt \right|^2$$

In this formula, a(t) is the acceleration of the particle as a function of time. It is non-zero through the period of time from 0 to τ. ν is the frequency at which the power emerges, so the frequency of emission is directly related to the timescale on which the acceleration changes.

If the acceleration is constant for a period of time τ, then the formula becomes:

$$P(\nu) \propto \left| \int_0^{\tau} e^{i(2\pi\nu)t} dt \right|^2$$

Which tends to zero if ντ is significantly greater than one and the fall-off of the acceleration is smooth on that timescale as well. If, on the other hand, the acceleration comes and goes on a timescale such that the exponential term does not have a chance to vary, then

Figure 8.4: The spectral output due to uniform acceleration for some period of time is uniform up to a cutoff set by 1/2πτ.

$$P(\nu) \propto \left| \int_0^{\tau} a(t) dt \right|^2$$

Which is a positive definite integral independent of ν. So, for uniform acceleration we have a uniform distribution of power up to the point where

$$\nu < \frac{1}{2\pi\tau}$$

As shown schematically in Figure 8.4

EXAMPLE

A capacitor discharges in 10μs, accelerating electrons out of their former equilibrium. At what frequency should we look for radiation?

Answer: $10\mu s = 10^{-5}$s. Therefore $\nu < (1/2\pi 10^{-5}) = 16$kHz. There will be a burst of radiation at all frequencies below 16kHz.

Normally we find the radiation at frequencies below the inverse of time across which the acceleration was applied.

EXERCISES

3. After being struck by the phasers of an enemy starship, the Enterprise is left with a net negative charge of 10^{19} electrons. The Enterprise charges directly at its enemy, accelerating smoothly from rest to .33c in 5 seconds.

a) How much energy does it radiate?

b) How much of this flux does the enemy see?

c) To avoid a collision the Enterprise ceases forward acceleration and starts the same acceleration perpendicular to its motion. What is the maximum flux the enemy could detect (in ergs/cm^2/s) at 10^5cm separation.

ν ν_c