### 9.1 THE CYCLOTRON FREQUENCY

Cyclotron radiation is to found when charged particles are trapped in a magnetic field. The Lorentz force acts perpendicular to both the magnetic field lines, creating an acceleration of charged particles that are moving perpendicular to the field. This acceleration, in turn, creates radiation.

The name derives from cyclotrons, the particle accelerators used by physicists since the 1930's to create highly energetic particles for study. The cyclotron makes use of the circular orbits that charged particles exhibit in a uniform magnetic field. Furthermore, the period of the orbit is independent of the energy of the particles, allowing the cyclotron to operate at a set frequency, and not worry about the energy of the particles at a given time.
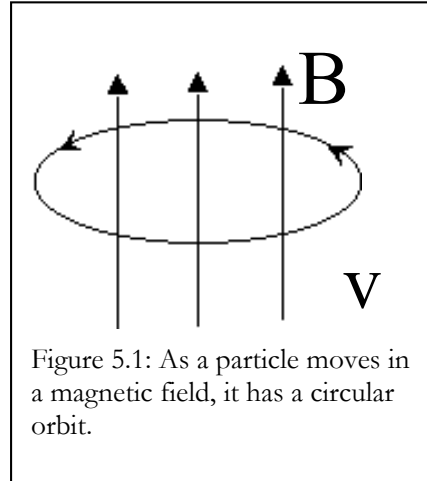
Consider a non-relativistic particle of mass m moving at velocity v perpendicular to the magnetic field lines. We can then write the equation for balance of centrifugal force against the Lorentz force:

Figure 5.1: As a particle moves in a magnetic field, it has a circular orbit.

$$\frac{mv^2}{r} = eB\frac{v}{c} \qquad (5.1)$$

Rearranging the equation we find:

$$\frac{v}{r} = \frac{eB}{mc} \qquad (5.2)$$

and recognizing that left side is an expression for angular frequency we find:

$$\omega_c = \frac{eB}{mc} \qquad (5.3)$$

and the period of the orbit is given by:

$$P = \frac{2\pi mc}{eB} \qquad (5.4)$$

The period of the orbit is thus inversely proportional to the magnetic field. Unlike planetary orbits, the period is independent of the energy of the orbiting particle or the size of its orbit.

From our earlier discussion of emission frequency, we expect that the cyclotron emission will occur near the frequency of the orbit (eB/2πmc). But in this case, it occurs exactly at the frequency of the orbit. Consider equation 4.xxx, which shows that the radiation is emitted at the Fourier Transform of the acceleration. The perfect circular motion of the particle leads to a perfect sinusoidal form to the acceleration and we have:

$$v \approx \int a(t)e^{-i\omega t}dt$$

(5.5)

We can write a(t) as a complex sinusoid to find:

$$v \approx \int e^{i\omega_c t}e^{-i\omega t}dt$$

(5.6)

Figure 5.2: A cyclotron emission spectrum consists of a spike at the fundamental frequency vc, and harmonics at integral factors higher.

which is a delta function at ω = ωc. In other words, the particles emit a narrow line of emission at the frequency of the cyclotron orbit.

$$v_c = \frac{eB}{2\pi m_e c}$$

(5.7)

$$v_c = \frac{eB}{2\pi m_e c} = \frac{4.8x10^{-10} B}{2\pi x0.91x10^{-27} x3x10^{10}} = 2.8Bx10^6 Hz = 2.8B \text{ MHz}$$

(5.8)

In Figure 5 we show the spectrum typical of cyclotron emission. It features a strong, narrow spike at vc, and smaller spikes at 2vc and 3vc. These smaller lines are the harmonics of the fundamental frequency.
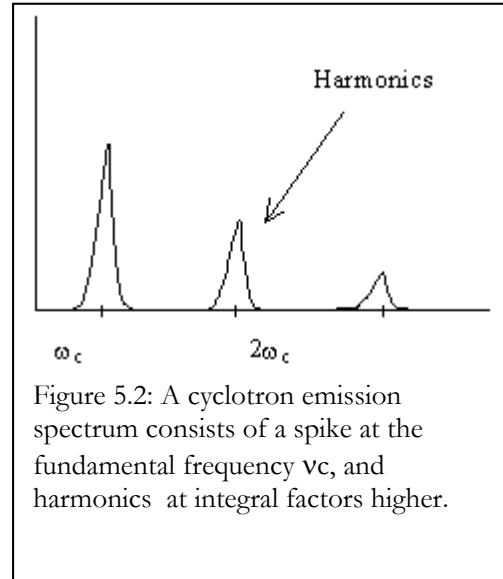
EXAMPLE

What is the frequency of emitted by plasma trapped in the 1000 Gauss magnetic field of Jupiter?

Answer: vc = 2.8x1000 MHz = 2.8GHz

Harmonics are the result of imperfections in the actual emission environment. The easiest place for radiation to be created that is not at vc is at an integral factor higher.

Just as lines appear at other frequencies, imperfections create a broadening of the line as well. The most obvious source of line broadening is non-uniformities in the magnetic field. Every magnetic field has variations in intensity as a function of position, and in astrophysical settings, it is difficult to create highly uniform fields. As an electron passes from one area of the field to another, its emission frequency will change with the field. Similarly, we usually have many electrons each emitting in response to the local field.

Other sources of broadening include a) collisional broadening from the electron failing to follow a perfect orbit, b) distortions of the emission caused interactions with the surrounding plasma, and c) relativistic effects if the charged particles are sufficiently energetic.

9.2 CYCLOTRON LOSSES

If we find a cyclotron source, its nature and the strength of its magnetic field are readily apparent, but further information can be gleaned from its intensity and variation with time. To understand what the intensity can tell us we need to calculate the radiation losses experienced by a charged particle trapped in a magnetic field.

We return to the Larmor Formula:

$$P = \frac{2e^2 a^2}{3c^3} \qquad (4.\text{xxxx})$$

But we can find an expression for the acceleration a using the Lorentz Force:

$$ma = e\frac{v}{c}B \qquad (5.9)$$

By substitution we find:

$$P = \frac{2e^4 v^2 B^2}{3c^5 m^2} \qquad (5.10)$$

Since v^2 is proportional to the kinetic energy of the particle, then power lost is proportional to the energy available.

$$\frac{dE}{dt} = \frac{-2e^4 v^2 B^2}{3c^5 m^2} = \frac{-4e^4 EB^2}{3c^5 m^3} \qquad (5.11)$$

$$\frac{dE}{E} = -\frac{4e^4 B^2}{3c^5 m^3} dt \qquad (5.12)$$

This simple differential equation has the solution:

$$E = E_0 e^{-\frac{4e^4 B^2}{3c^5 m^3}t} = E_0 e^{-\frac{t}{t_0}} \qquad (5.13)$$

where

$$t_0 = \frac{3c^5 m^3}{4e^4 B^2} = \frac{3x(3x10^{10})^5 x(0.91x10^{-27})^3}{4x(4.8x10^{-10})^4 xB^2} = \frac{2.6x10^8}{B^2} \text{ seconds} \approx 10\text{years}/B^2 \quad (5.14)$$

Plasma confined in a field on the order of one Gauss or less can thus last quite a long time before decaying completely due to cyclotron emission. It should also be noted that since the timescale for decay scales as the cube of the mass, the protons in the plasma would take a billion times longer to lose their energy. Instead, the electrons due the radiating, and the protons transfer their energy over to electrons faster than they can radiate the energy directly. Thus we do not expect to see emission from the protons in the plasma.

EXAMPLE

The interstellar medium is permiated by a magnetic field of about 10^{-6} Gauss. How long can plasma exist before it radiates away its energy through cyclotron radiation?

Answer: $t_0 = 10\text{years} / B^2 = 10\text{years}/10^{-12} = 10^{13}$ years. This is greater than the age of the universe, so cyclotron is not a significant cooling mechanism for interstellar gas.

EXERCISES:

1. In 1977 the German X-ray astronomer J. Trumper observed the neutron star Hercules X-1. He found two emission lines, one at 55keV and the other at 110keV. He interpreted these as cyclotron emission from charged particles near the surface of a magnetized neutron star.
   a) Derive the strength of the magnetic field at the surface of the neutron star.
   b) What is the velocity of the electrons assuming they have a temperature of 108K? What is the radius of their orbit?
   c) What is the timescale for the decay of the electron orbit on this neutron star?
   d) At what frequency would the protons be emitting their cyclotron lines? What is the corresponding wavelength?

2. Some radio astronomers observe a flare in a star known to be 30pc from the Sun. It fades exponentially with a timescale of 5 minutes. They believe it is cyclotron radiation.
   a) What was the strength of the magnetic field in the flaring plasma?
   b) At what frequency should the emission be found?
   Remembering that the volume of a flare increases as it fades, describe qualitatively the shape and time behavior of the radio spectrum.