SgrA*

The animation of infrared observations by Eckart, Genzel et al. (2002) at http://www.mpe.mpg.de/www_ir/GC/ shows stars buzzing like bees around the position of SgrA*, the unresolved point radio source thought to mark the black hole Sagittario at the center of our Galaxy. In a ‘virialized’ gravitating system like this (meaning one that has reached an equilibrium where it is neither collapsing nor expanding), the average velocity $v$ of stars at radius $r$ from the center is

$$v = \sqrt{\frac{GM}{r}}$$ (1)

where $M$ is the enclosed mass interior to $r$, including both black hole and stars. [Incidentally, this is Kepler’s third law of planetary motion, in disguise.]

Astronomers deduce the distances $r$ of stars from SgrA* from their angular separation on the sky multiplied by the known distance 8,000 pc, eight thousand parsecs, to the Galactic center. A parsec is a unit of distance, equal to about 3 lightyears.

1. What two methods do you think astronomers could use to determine the velocity $v$ of a star, such as those in the animation, relative to SgrA*?

2. What aspect of the curve of enclosed mass $M$ versus radius $r$ suggests that there is a black hole at the Galactic center? What is the mass of the black hole, according to the curve?

3. A star of mass $M_*$ and radius $R_*$ will be tidally torn apart if the density interior to its distance $r$ from the black hole exceeds the density of the star, i.e. if

$$\frac{M}{r^3} > \frac{M_*}{R_*^3}.$$ (2)
[This is different from the condition that you will be torn apart: a star is held together by gravity, whereas you are held together by electric forces.] At what distance \( r \) from SgrA*, in stellar radii \( R_* \), will a 1 M\(_\odot\) star be torn apart? What is the tear-apart distance, in parsecs, if the star were similar to the Sun, with radius \( R_*=R_\odot = 2 \times 10^{-8} \) pc? What is the tear-apart distance, in parsecs, if the star were similar to Betelgeuse, a red supergiant with about the same mass as the Sun, but with radius \( R_*=500 R_\odot = 10^{-5} \) pc? Compare these tear-apart distances to the distance of closest approach, in parsecs, of the star S2, plotted as the innermost point on the curve of enclosed mass \( M \) versus radius \( r \).

4. The curve of enclosed mass \( M \) versus radius \( r \) at large radii fits approximately the power-law relation (the dot-dash line on the graph)

\[
M_{\text{stars}} = 1.5 \times 10^6 M_\odot \left( \frac{r}{1 \text{ pc}} \right)^{1.2}
\]  

(3)

which can be interpreted as the mass \( M_{\text{stars}} \) of stars closer to the black hole than \( r \). The animation shows a region of radius about 10 lightdays, equivalent to about \( r = 0.01 \) pc (a hundredth of a parsec). According to the fit (3), \textbf{roughly} how many stars might you expect there to be in the region shown by the animation? [Hint: It is reasonable to suppose that the average mass of a star is about 1 M\(_\odot\), one solar mass.] What might be the reason that the actual number of stars in the animation is much less than this?

5. Given the information you have accumulated, do you think that the star S2 is more likely to be a star like the Sun, or a red giant star? Why? [Stars like the Sun are much more common than red giants, but red giants are much more luminous than the Sun.]

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Figure 1: http://www.mpe.mpg.de/www_ir/GC/
Figure 2: Schödel et al. 2002, Nature 419, 694.