

ASTR 3740 Relativity & Cosmology Spring 2004. Problem Set 7.
Due Fri 28 Apr

1. Horizon at Recombination

What angle does the horizon at Recombination subtend on the CMB today? Assume a flat, matter-dominated Universe. Express your answer first in terms of the redshift factor $1 + z_R$ of Recombination, and then translate your answer into degrees for the case $1 + z_R \approx 1300$. [Hint: This is simpler than the realistic case of a flat Universe with a cosmological constant, but it turns out that the cosmological constant only changes the angle a bit; the angle mostly depends on the curvature. In a flat, matter-dominated Universe, the comoving distance to the horizon at a time when the cosmic scale factor is a and the Hubble parameter is H is

$$x = \frac{2c}{aH} . \quad (1.1)$$

The angle, in radians, subtended by the horizon at Recombination is the ratio of the comoving horizon size x_R at Recombination to the comoving horizon size x_0 now:

$$\text{Angle} = \frac{x_R}{x_0} \quad (1.2)$$

(the formula is this simple because the geometry is flat). Recall that $H = \dot{a}/a$, and from Problem Set 6 that in a flat, matter-dominated Universe

$$a \propto t^{2/3} . \quad (1.3)$$

The redshift at recombination is

$$1 + z_R = \frac{a_0}{a_R} . \quad (1.4)$$

Remember that there are 180 degrees in π radians.]

2. Horizon Problem

(a) Expansion factor

The temperature of the CMB today is $T_0 \approx 3\text{K}$. By what factor has the Universe expanded (i.e. what is a_0/a) since the temperature was the Planck temperature $T \approx 10^{32}\text{K}$? [Hint: you already did this in Problem Set 6, Question 3c.]

(b) Hubble distance

By what factor has the Hubble distance c/H increased during the expansion of part (a)? Assume that the Universe has been mainly radiation-dominated during this period, and that the Universe is flat. [Hint: For a flat Universe $H^2 \propto \rho$, and for radiation-dominated Universe $\rho \propto a^{-4}$.]

(c) Comoving Hubble distance

Hence determine by what factor the comoving Hubble distance $x_H = c/(aH)$ has increased during the expansion of part (a).

(d) Comoving Hubble distance during inflation

During inflation the Hubble distance c/H remained constant, while the cosmic scale factor a expanded exponentially. What is the relation between the comoving Hubble distance $x_H = c/(aH)$ and cosmic scale factor a during inflation? [You should obtain an answer of the form $x_H \propto a^?$.]

(e) e -foldings to solve the Horizon Problem

By how many e -foldings must the Universe have inflated in order to solve the Horizon Problem? Assume again, as in part (a), that the Universe has been mainly radiation-dominated during expansion from the Planck temperature to the current temperature, and that this radiation-dominated epoch was immediately preceded by a period of inflation. [Hint: Inflation solves the Horizon Problem if the currently observable Universe was within the Hubble distance at the beginning of inflation, i.e. if the comoving $x_{H,0}$ now is less than the comoving Hubble distance $x_{H,i}$ at the beginning of inflation. The ‘number of e -foldings’ is $\ln(a_f/a_i)$, where \ln is the natural logarithm, and a_i and a_f are the cosmic scale factors at the beginning (i for initial) and end (f for final) of inflation.]

3. Relation between Horizon and Flatness Problems

Show that Friedmann’s equation can be written in the form (compare Problem Set 6, Question 3a)

$$\Omega - 1 = \kappa x_H^2 \tag{3.1}$$

where $x_H \equiv \frac{c}{aH}$ is the comoving Hubble distance. Use this equation to argue in your own words how the horizon and flatness problems are related. [The main part of this question is not the math but the explanation. You should convince the grader that you understand physically what is going on.]