ASTR 3740 Relativity & Cosmology Spring 2004. Project 3. Fri Apr 16.

The power spectrum of the CMB

This project is a highly over-simplified version of how to interpret the power spectrum of the Cosmic Microwave Background (CMB). For a more accurate description, see Wayne Hu and Martin White "The Cosmic Symphony", Scientific American, Feb 2004 issue, or Wayne Hue and Scott Dodelson (2002) Annual Reviews of Astronomy & Astrophysics.

The attached graph shows the power spectrum of Cosmic Microwave Background fluctuations from the WMAP satellite. The data are from `http://lambda.gsfc.nasa.gov/product/map/m_products.cfm`.

1. Sketch a cosine curve
$$T(t) \propto \cos(2\pi\nu t) \tag{1}$$
starting from time $t = 0$, the moment of the Big Bang. Here $T(t)$ represents the temperature fluctuation of a standing sound wave in the photon-baryon fluid before Recombination, and ν is its frequency.

2. Consider the proposition: Sound waves of different frequencies evolve independently. True or false? What evidence can you draw from your personal experience to support your answer?

3. We see the imprint of a sound wave in the CMB sky at the time of Recombination, $t = t_{\rm Rec}$. The frequency ν of the sound wave is related to its wavelength λ by
$$\nu = \frac{c_s}{\lambda} \tag{2}$$
where $c_s \approx c/\sqrt{3}$ is the approximate speed of sound in the photon-baryon fluid (the speed of light divided by the square root of three). On the sky, the wavelength λ subtends an angle θ given by
$$\lambda = \theta D \tag{3}$$
where D is the distance to the CMB. Finally, the angle θ is related to the angular harmonic number l by
$$\theta = \frac{2\pi}{l} \ . \tag{4}$$
Recast the expression (1) for the temperature fluctuation $T(t)$ in terms of l, D, c_s, and $t_{\rm Rec}$.

4. Rewrite your expression for $T(t)$ as
$$T(t) \propto \cos(\pi l / l_{\rm pk}) \tag{5}$$
with $l_{\rm pk}$ given in terms of D, c_s, and $t_{\rm Rec}$.

5. At what harmonics l will the temperature fluctuation be zero? At what harmonics l will the temperature fluctuation be largest (either positive or negative)?

6. Physically, why do you think we reexpressed the expression for $T(t)$ in terms of the harmonic number l rather than leaving it in terms of the angle θ on the sky? [Hint: Harmonics l are pure modes of vibration on a sphere.]

7. The correspondence between frequency ν and angular harmonic l is good but not quite perfect. Why? [Hint: A sound wave can have a component along the line of sight as well as transverse to the line of sight.]

8. Make an approximate correspondence between the peaks and troughs predicted by your simplified model and those observed with WMAP. [Hint: No, it ain't perfect, but that's because our model is over-simplified. You probably notice that the amplitudes of the peaks and troughs show interesting differences, which are not in our simple model.]

9. If the Universe is spatially curved, then it shifts the positions of the peaks in the CMB power spectrum. Which way, and why (qualitatively)? [Hint: Don't bother with the mathematics, because we haven't set it up, and the details are not entirely trivial. Sketch a picture of a closed Universe, and think about how the angular size of a fixed yardstick at a fixed geodesic distance from us is changed by the curvature.]

10. A feature of inflation is that it generates only cosine type fluctuations, as specified by equation (1). These are pure curvature fluctuations. There are also sine type fluctuations $T(t) \propto \sin(2\pi\nu t)$, called isocurvature fluctuations. If there were isocurvature fluctuations, how would that change the power spectrum of CMB fluctuations?

Complications

Ok, so I have to admit that the real CMB is quite a bit more complicated than suggested above.

First, the time t and wavelength λ discussed above are really **comoving** quantities in the expanding Universe. The wavelength λ of a sound wave expands with the Universe, and has a fixed comoving size, not a fixed proper size. The time t is not the proper cosmic time, but rather what is called the **conformal time**, which has the definining property that light moves one unit of comoving distance per one unit of conformal time.

Next, the temperature fluctuation $T(t)$ is not really a temperature fluctuation, but rather the sum $[T + \Psi](t)$ of a temperature fluctuation T and a gravitational potential fluctuation Ψ. In a normal sound wave, where the wave is compressed, the pressure (T) provides a restoring force that tends to make the wave expand again. Gravity (Ψ) however counters the restoring force of pressure, tending to make the compression want to compress even more. There is a balance between pressure and gravity when $T + \Psi = 0$, so sound waves oscillate about $T + \Psi = 0$ rather than about $T = 0$.

To confuse matters, the gravity actually wins out over the pressure in the initial $t = 0$ conditions (specifically, $\Psi = -(3/2)T$), so positive T in our simplified analysis actually corresponds to low temperature and pressure. The simplified T is actually more like a potential. You can think of the wave as falling from the top to the bottom of a potential well as it undergoes its first half oscillation.

Please write your verbal answers on this sheet.

Scribe's name:

Names of other members of the group: