I. SPECIAL RELATIVITY

Physicist: describes
Philosopher: asks why?
People : do both

Structure of spacetime

(a) Space 3D } form a 4D manifold, a topological space which looks locally like $\mathbb{R}^4$
Time 1 D

Why?
What if space were 1 D ?
7 D ?
What if time were 2 D ?

(b) Space & time intervals can be measured
space – rulers
time – clocks

How do clocks know to tick at a certain rate?

In everyday life, different observers obtain reproducible measurements of space & time.
Why (not)?

(c) Does spacetime have an existence independent of objects (observers) in it?

- Yes – Galileo Galilei (1564 – 1642)
- " – Isaac Newton (1643 – 1727)
- No – Einstein

Do fundamental particles know about the structure of spacetime?
Apparently yes!
In quantum mechanics, particles have
- frequency ν $\quad E = h\nu$
- wavelength λ $\quad p = h/\lambda$
- spin so they act ~~like~~ as if they were gyroscopes: they know about direction in space.

Inertial spacetime frames

Newton's 1st law:
"Body moves in a straight line at constant velocity, unless acted on by forces."

A system of spacetime coordinates with respect to which unaccelerated bodies move in straight lines at constant velocity is called an $\boxed{\text{inertial frame}}$.

Can you think of examples of non-inertial frames?

- Globally inertial frame
  is inertial frame constructed over all spacetime. Existence thereof postulated by
  - Galileo/Newton
  - Special Relativity.

  Existence equivalent to assumption that spacetime has 4D Euclidean geometry.

  Abandoned by General Relativity, which postulates existence instead of
- Locally inertial frames.
  In a sufficiently small neighborhood of each spacetime point, there exists a coordinate system with respect to which unaccelerated bodies move in straight lines.

Symmetry

Arguably the single most important concept in all of modern physics.

Symmetry = laws of physics are unchanged (values of observable quantities are unchanged) when system is transformed in some way (e.g. translated, rotated, etc.).

Global symmetry

= Transform everything everywhere by the same amount.

Consequence (Noether's theorem): corresponding to every global symmetry is a conservation law.

Local symmetry (applies to continuous, not discrete symmetries)

= Transform

- everything at same spacetime point by the same amount;
- things at different points by different amounts.

Consequence: Forces!

Spacetime symmetries

Symmetry	Dimensions	Consevred quantity
Translation in time	1D	Energy
Translation in space	3D	Momentum
Rotation in space	3D	Angular momentum
Velocity boost	3D	Velocity of center of mass
Time reversal $t \to -t$	Discrete	T
Spatial inversion $\mathbf{r} \to -\mathbf{r}$	"	P (Parity)

Other symmetries

Particle exchange	Discrete	system is { bosonic (integral ang. mom) / fermionic ($\frac{1}{2}$-integral ang. mom.)
Particle $\leftrightarrow$ antiparticle (or "Charge conjugation")	Discrete	C
Phase of wavefunction of charged particle	1D	Electric charge
⋮		

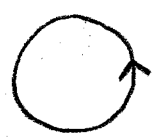
Can you think of a spacetime transformation which is not a good symmetry?

Symmetries & Forces

Symmetry	Force	"Strength"
Space-time symmetries	Gravity	$\sim 10^{-42}$
$U_{em}(1)$	Electromagnetism	$1/137$
$SU(2) \times U_Y(1)$	Electroweak	$\sim 10^{-4}$
$SU(3)$	Color	~ 1

Notes:

1. "Strength" means the relative strength of the force between two protons in a nucleus. A more precise definition can be made.
2. $\underbrace{U(1)}_{\text{unitary group of 1 dimension}}$, $\underbrace{SU(2)}_{\text{special unitary group of 2 dimensions}}$ etc denote certain compact groups.

   e.g. $U(1)$ is the group of rotations of a circle: 
3. The electroweak symmetry $SU(2) \times U(1)$ appears only at energies $\gtrsim$ 100 GeV (10^{15} K). At lower energies the electroweak symmetry is "spontaneously broken" into the electromagnetic and weak force. The strength of 10^{-4} is the strength of the broken weak force; at high energy, the strengths of the electromagnetic & weak forces become the same ($\sim 10^{-2}$).

Spatial 3-vectors

Vector = quantity with magnitude & direction.

3D vector notation is wonderful shorthand way to express & deal with spatial symmetries.

$$\mathbf{r} = (x, y, z)$$

vector — coordinates in one frame A

$$\mathbf{r}' = (x', y', z')$$

coordinates in another frame B

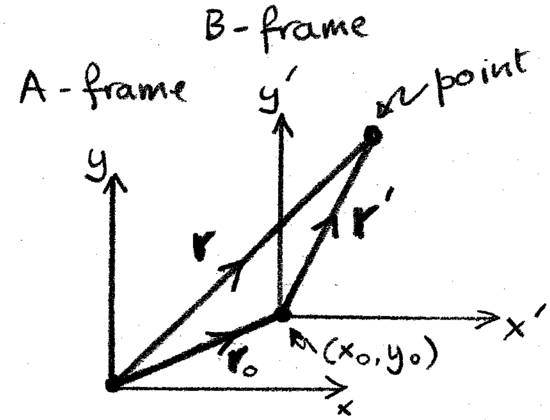
Vector notation recognizes that there is a certain arbitrariness (because of existence of symmetries!) in choice of coordinates.

Crucial point:

If $\mathbf{r} = \mathbf{s}$ in one frame

then $\mathbf{r}' = \mathbf{s}'$ in any other

Spatial translation



Frame $\mathcal{B}$ is translated by

$$\mathbf{r}_0 = (x_0, y_0, z_0)$$

with respect to A.

Then point

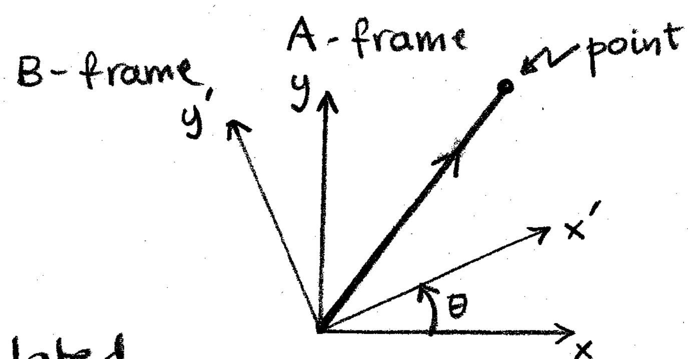
$$\mathbf{r} = (x, y, z) \quad \text{in A frame}$$

has coordinates

$$\mathbf{r}' = (x', y', z') = (x - x_0, y - y_0, z - z_0) \quad \text{in B frame}$$

ie $\boxed{\mathbf{r}' = \mathbf{r} - \mathbf{r}_0}$

Spatial rotation



Frame B rotated by θ about z axis (for definiteness).

Then coordinates in B and A frames related by

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$
$$z' = z$$

which may also be written

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

or equivalently

$\boxed{\mathbf{r}' = \mathbf{O}\mathbf{r}}$ an "orthogonal" matrix, $\mathbf{O}^T = \mathbf{O}^{-1}$

Galilean velocity transformation

Notice that Newton's 3rd law

$$\mathbf{F} = m \frac{d^2\mathbf{r}}{dt^2}$$

is independent of both

- origin of coordinate system
- velocity of coordinate system

Crucial Galilean assumption:
two systems in relative motion experience the same universal time, ie can assume (once synchronized)

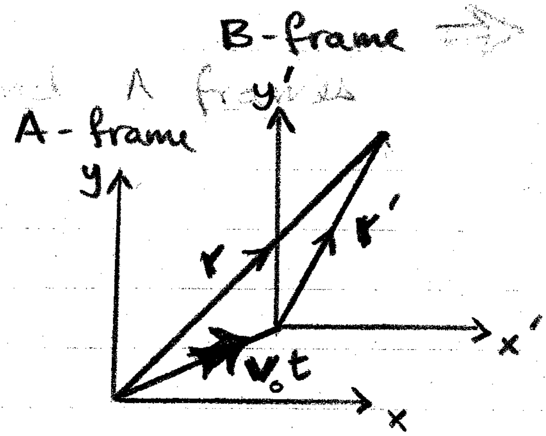
$$\boxed{t' = t}$$

If origin $\mathbf{r}_0$ of B relative to A frame moves at constant velocity $\mathbf{v}_0$, so

$$\mathbf{r}_0 = \mathbf{v}_0 t$$

then coordinates in B & A frames are related by

$$\boxed{\mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t}$$



Galilean law of velocity addition

Velocity of a point is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \quad \text{in A system}$$

$$\mathbf{v}' = \frac{d\mathbf{r}'}{dt} \quad \text{in B system}$$

ie $$\boxed{\mathbf{v}' = \mathbf{v} - \mathbf{V}_0}$$

Velocities add.

<u>Crucial</u> here that $\Delta t' = \Delta t$