ASTR 3740 Relativity & Cosmology Spring 2007. Problem Set 7.
Due Wed 2 May

1. Horizon at Recombination

What angle does the horizon at Recombination subtend on the CMB today? Assume for simplicity a flat, matter-dominated Universe. Express your answer first in terms of the redshift factor $1 + z_R$ of Recombination, and then translate your answer into degrees for the case $1 + z_R \approx 1100$. [Hint: This is simpler than the realistic case of a flat Universe with a cosmological constant, but it turns out that the cosmological constant only changes the angle a bit; the angle mostly depends on the curvature. In a flat, matter-dominated Universe, the comoving distance to the horizon at a time when the cosmic scale factor is a and the Hubble parameter is H is

$$x = \frac{2c}{aH} \ . \tag{1.1}$$

The angle, in radians, subtended by the horizon at Recombination is the ratio of the comoving horizon size x_R at Recombination to the comoving horizon size x_0 now:

$$\text{Angle} = \frac{x_R}{x_0} \tag{1.2}$$

(the formula is this simple because the geometry is flat). Recall that $H = \dot{a}/a$, and from Problem Set 6 that in a flat, matter-dominated Universe

$$a \propto t^{2/3} \ . \tag{1.3}$$

The redshift at recombination is

$$1 + z_R = \frac{a_0}{a_R} \ . \tag{1.4}$$

Remember that there are 180 degrees in π radians.]

2. Horizon Problem

(a) Expansion factor

The temperature of the CMB today is $T_0 \approx 3\,\text{K}$. By what factor has the Universe expanded (i.e. what is a_0/a) since the temperature was the Planck temperature $T \approx 10^{32}\,\text{K}$? [Hint: How is temperature T related to cosmic scale factor a? You did this before in Problem Set 6, Question 3.]

(b) Hubble distance

By what factor has the Hubble distance c/H increased during the expansion of part (a)? Assume that the Universe has been mainly radiation-dominated during this period, and that the Universe is flat. [Hint: For a flat Universe $H^2 \propto \rho$, and for radiation-dominated Universe $\rho \propto a^{-4}$.]

(c) Comoving Hubble distance

Hence determine by what factor the comoving Hubble distance $x_H = c/(aH)$ has increased during the expansion of part (a).

(d) Comoving Hubble distance during inflation

During inflation the Hubble distance c/H remained constant, while the cosmic scale factor a expanded exponentially. What is the relation between the comoving Hubble distance $x_H = c/(aH)$ and cosmic scale factor a during inflation? [You should obtain an answer of the form $x_H \propto a^?$.]

(e) e-foldings to solve the Horizon Problem

By how many e-foldings must the Universe have inflated in order to solve the Horizon Problem? Assume again, as in part (a), that the Universe has been mainly radiation-dominated during expansion from the Planck temperature to the current temperature, and that this radiation-dominated epoch was immediately preceded by a period of inflation. [Hint: The 'number of e-foldings' is $\ln(a_f/a_i)$, where ln is the natural logarithm, and a_i and a_f are the cosmic scale factors at the beginning (i for initial) and end (f for final) of inflation. Inflation solves the Horizon Problem if the currently observable Universe was within the Hubble distance at the beginning of inflation, i.e. if the comoving $x_{H,0}$ now is less than the comoving Hubble distance $x_{H,i}$ at the beginning of inflation.]

3. Relation between Horizon and Flatness Problems

Argue that the first Friedmann equation can be written in the form

$$\Omega_K = -\kappa x_H^2 \tag{3.1}$$

where κ is the curvature constant, $\Omega_K = 1 - \Omega$ is the curvature density, and $x_H = c/(aH)$ is the comoving Hubble distance. Use this equation to argue in your own words how the horizon and flatness problems are related. [The main part of this question is not the math but the explanation. You should convince the grader that you understand physically what is going on.]