**ASTR 5540 Math Meth Fall 2008. Problem Set 9. Due Mon Nov 3**

## 1. 1-D PM code

In the language of your choice (IDL, Mathematica, c, ...), write a 1-dimensional gravitational Particle-Mesh (PM) code. Choose whatever units of length, mass, and time make programming easiest.

### (a) Functions

Your code should contain functions that implement the following:

1. A cloud-in-cell (CIC) smoothing window $W(x)$;
2. Wavenumber of a Fourier mode;
3. Mesh density given particle positions;
4. FFT of mesh density;
5. Potential given density, in Fourier space;
6. Acceleration given potential, in Fourier space;
7. FFT acceleration back on to mesh;
8. Acceleration at particle positions given mesh acceleration.

[Hints: Function 2 depends on how the FFT you use stores the FT. In mathematica for example, the FT is stored as a complex array $\{\tilde{a}_0, \tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_{[\frac{N}{2}]}, \tilde{a}_{[\frac{N}{2}]+1-N}, ..., \tilde{a}_{-2}, \tilde{a}_{-1}\}$, so that the wavenumbers are $\{0, 1, 2, ..., [\frac{N}{2}], [\frac{N}{2}]+1-N, ..., -2, -1\}$. FFTs that input a real array typically return $\{\tilde{a}_0, \mathrm{Re}\,\tilde{a}_1, \mathrm{Im}\,\tilde{a}_1, ..., \mathrm{Re}\,\tilde{a}_{[\frac{N}{2}]}\}$, in which case the wavenumbers are $\{0, 1, 1, ..., [\frac{N}{2}]\}$. Function 5 should set the zero'th Fourier mode of the potential to zero, $\tilde{\phi}_0 = 0$. Function 6 should set the zero'th and $[\frac{N}{2}]$'th (for even $N$) Fourier mode of the acceleration to zero, $\tilde{g}_0 = \tilde{g}_{[\frac{N}{2}]} = 0$. The sign of the acceleration $\tilde{g}$ in Fourier space depends on the phase convention of the FFT; for example mathematica's phase convention is minus the convention adopted in class.]

### (b) Leap-frog integrator

Implement a leap-frog integrator for the positions $\boldsymbol{x}$ and velocities $\boldsymbol{v}$ of particles:

$$\begin{aligned} \boldsymbol{v}_{i+\frac{1}{2}} &= \boldsymbol{v}_{i-\frac{1}{2}} + \boldsymbol{g}_i \, \Delta t \ , \\ \boldsymbol{x}_{i+1} &= \boldsymbol{x}_i + \boldsymbol{v}_{i+\frac{1}{2}} \, \Delta t \ , \end{aligned} \tag{1.1}$$

where $\Delta t$ is a suitable time step.

### (c) Integrate and plot

Choose a reasonable mesh-size $N$. Choose a small number of particles, say 2 to 4. Give the particles random initial positions, and zero initial velocities. Choose a time step $\Delta t$. Integrate for a sufficient number of steps, at least 100. Plot the positions of the particles as a function of time $t$.