Implications of cognitive studies for teaching physics

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I. INTRODUCTION

Many of us who have taught introductory physics for many years recall with dismay a number of salient experiences: a reasonably successful student who can produce a graph but cannot say what it means; a top student who can solve all the problems but not give an overview or simple derivation; many students of varying abilities who memorize without understanding despite our most carefully crafted and elegant lectures.

As physics teachers who care about physics, we have a tendency to concentrate on the physics content we are teaching. We often are most concerned for those students who are like we were—that small fraction of our students who find physics interesting and exciting and who will be the next generation of professional physicists. But the changes in our society and in the role of technology for the general public mean that we must change the way we are teaching. It no longer suffices to reproduce ourselves. Society has a great need not only for a few technically trained people, but for a large group of individuals who understand science.

During the past decade, data have built up that demonstrate that as physics teachers we fail to make an impact on the way a majority of our students think about the world.[1–5] We have readjusted our testing so that the students can succeed and we have then either fooled ourselves that we are teaching them successfully or lowered our standards by eliminating understanding from our definition of successful learning. Alan van Heuvelen[6] has remarked that in his study of a typical introductory lecture class, 20% of the students entered the first semester of an introductory calculus-based physics class as Newtonian thinkers. The impact of the course was to increase that number to 25%. If we want to reach a substantial fraction of our students, we must pay much more attention to how students learn and how they respond to our teaching. We must treat the teaching of physics as a scientific problem.

A few physicists have begun to perform detailed experiments to determine what our students are thinking and what works in the teaching of physics. Some of their articles are of tremendous importance and I believe should be read by every physics teacher (see Refs. 4 and 5, and references therein). But even among these few articles, only a small fraction of the authors attempt to place their results in a general theoretical framework—to give us a way of thinking about and organizing the results.[7] Those of us in physics know well that advancement in science is a continual dance between the partners of theory and experiment, first one leading, then the other. It is not sufficient to collect data into a "wizard's book" of everything that happens. That's not science. Neither is it science to spout high-blown theories untainted by "reality checks." Science must build a coherent and clear picture of what is happening at the same time as it continually confirms and calibrates that picture against the real world.

The time has come for us to begin the development of a framework for understanding and talking about student learning. Some of the results of the past few decades in cognitive studies[8] begin to provide such a framework.

Cognitive studies focuses on how people understand and learn. It is still an amorphous field, and it is not yet really a single discipline. It overlaps many areas from anthropology to neurophysiology. It may not yet be "a science" as we in physics use the term, but developments in the past few decades have changed drastically what we know about how the mind works.

The issue of how to teach physics is a difficult one: the attempt of a naive student to build a good understanding of physics involves many intricate processes over a long period of time. These processes tend to be much more complex than those most cognitive scholars have addressed.[9] Nonetheless, some of the basic ideas of cognitive studies appear to be both firmly grounded and useful to the teacher of physics.

In this essay I briefly review some of the lessons I have learned from cognitive studies. This is not a review article, but a narrow selection from a small slice of a large field. For those interested in a more substantial introduction to cognitive studies I recommend Howard Gardner's historical overview,[10] the collection of articles assembled by Gentner and Stevens,[11] and some of the articles in the reprint volume collected by Collins and Smith.[12] These will give an entry point into the modern cognitive literature. The book by Inhelder and Piaget[13] has lots of discussion of experiments on how adolescents learn physics. Some articles by leading educational specialists also can help link to the existing literature.[14] Just for fun, I have to add Donald Norman's delightful book on how people interact with objects in their world.[15] For an introduction to the physics education research literature, Arnold Arons's book[5] and a few review articles[16] provide an appropriate entry point.

I have grouped what I have learned from the cognitivists into four broad principles with elaborative corollaries. One of the things students of cognitive processes have learned about thinking is that it is fuzzy. The sharp, crisp operations of formal logic or the digital computer are not appropriate models for the way most people think. Therefore, it is not correct to call these principles "theorems" or "laws of cognitive science." Nor is it correct to use them as such. They cannot provide us with hard and fast rules for what to do. Using them incautiously without reference to experimental data can lead us to the wrong conclusions. But I have found that they help me to organize my thinking about my students and to refocus my attention. Instead of concentrating only on the organization of the physics content, I now also pay attention to what my students are doing when they interact with a physics course. This is not to suggest that an emphasis on content is somehow unimportant or should be neglected. What we are teaching is important, but it must be viewed in the context of what our students learn.

II. BUILDING PATTERNS: THE CONSTRUCTION PRINCIPLE

The fundamental change that has led to the breakthroughs in cognitive studies in the past few decades has been a new willingness to model what is happening in the mind in terms of inferred structures. For the first half of this century,[10] studies of thinking were severely constrained by the "behaviorist" philosophy that one should formulate all one's theories only in terms of direct observables. This is like the S-matrix theory of elementary particles which insisted that we should only formulate our theories in terms of observable particles and their scattering amplitudes. Elementary particle physicists only made their breakthrough when they were willing to formulate their ideas in terms of quarks and gluons—particles which could only be inferred and not be seen directly. Cognitive scholars started to make real progress when they began to be willing to formulate how people were thinking in terms of mental patterns or models that could not be directly observed or measured.

Principle 1: (Weak form) People tend to form mental patterns: This is the fundamental hypothesis about how the mind works. On some levels, there is direct observation of this mental processing by patterning. For example, it has been demonstrated in detail that we process visual information on a variety of levels to form patterns beginning with the first layer of nerve cells attached to the retina, and the process continues through many stages deep into the brain.[17] I once attended a physics colloquium given by Jerome Lettvin on the subject of blind spots in the visual field. He passed out the standard blind-spot demonstration pages[18] that let us clearly find the blind spot in our eye. We then moved the end of a pencil into our blind spots and saw the end of the pencil disappear as if bitten off. Yet there was no "blank spot" in the visual field. The brain fills in the background—here the simple white of a blank page. But it will fill in even a quite complex pattern. If the page had been covered with plaid, my brain would still have filled in my blind spot with the appropriate pattern. But note that the patterning was not sufficiently sophisticated to produce the "right" answer! My automatic filling led to my seeing paper in the blind spot, not the rest of the pencil.

The tendency of the human mind to form patterns is not just limited to the analysis of sensory data. This leads me to state the principle in a stronger (and more relevant) form.

Principle 1: (Strong form) People tend to organize their experiences and observations into patterns or mental models.

I use the term *mental model* for the collection of mental patterns people build to organize their experiences related to a particular topic. I use the term *schema* (pl. schemas or schemata) to describe the basic elements of these mental models. I think of a schema as a "chunk" or "object" (in the sense of object-oriented programming). It is a collection of closely linked data together with some processes and rules for use. Be careful of the use of the word "model." It tends to convey something clockwork—a mechanism that has links and rules and operates in a well-defined way. These are not the characteristics of many mental models.

The characteristics of mental models and schemas are still vigorously debated.[19] Despite attempts to build a general representational system for mental models, none has yet been widely accepted. However, some results are clear.[20]

Properties of mental models

Mental models have the following properties:

(1) They consist of propositions, images, rules of procedure, and statements as to when and how they are to be used.
(2) They may contain contradictory elements.
(3) They may be incomplete.
(4) People may not know how to "run" the procedures present in their mental models.
(5) Elements of a mental model do not have firm boundaries. Similar elements may get confused.
(6) Mental models tend to minimize expenditure of mental energy. People will often do extra physical activities—sometimes very time consuming and difficult—in order to avoid a little bit of serious thinking.

This inferred structuring of mental models is distinctly different from what we usually assume when teaching physics. We usually assume that our students either know something or they do not. The view of mental models we learn from cognitive scholars suggests otherwise. It suggests that students may hold contradictory elements in their minds without being aware that they contradict.

I had an interesting experience that illustrated for me vividly the surprising fact that one's mental model may contain contradictory elements. Ron Thornton visited the University of Maryland a few years ago to give a seminar on his now famous work on using the Sonic Ranger to teach the concept of velocity.[3] The Ranger detects the position of an object using sonar and can display the position or the velocity of the detected object on a computer screen in live time. Thornton set up the Ranger to display velocity and had the computer show a preset pattern (a square wave). He then called me up to the front of the room to serve as a guinea pig and try to walk so my velocity matched the preset pattern.

I had no hesitation in doing this. I had been teaching physics for nearly 20 years and felt perfectly comfortable with the concept of velocity. I did my first trial without thinking; I walked backward until my velocity reached the height of the preset square wave. Then I stopped and my velocity dropped to zero immediately! I asked for another chance, and this time, putting my brain in "velocity mode," I was able to reproduce the curve without difficulty.

What this experience said to me was that, for normal walking, I still maintained a naive (but appropriate!) position-dominated proposition in my mental model of motion. I also had a correct proposition for the concept of velocity, but I had to consciously apply a rule telling me to use it.

I have also had personal experiences illustrating characteristic 6. I once spent an hour searching through my hard drive and piles of floppy disks to find a short paragraph (three sentences!) that I needed again. When I found it, I realized it would have taken me only five minutes to have rewritten it from scratch. A nice example of this characteristic is Donald Norman's study of how people use complex calculators (Ref. 20).

An important aspect of Principle 1 is that "people ... *organize* their experiences into ... mental models" with the emphasis on the fact that people must build their *own* mental models. This is the cornerstone of the educational philosophy known as *constructivism*. For this reason, I refer to Principle 1 as "The Construction Principle."

An extreme[21] statement of constructivism is:

You cannot teach anybody anything. All you can do as a teacher is to make it easier for your students to learn.

Of course, facilitation can be critical to the learning process. Constructivism should not be seen as disparaging teaching, but as demanding that we get feedback and evaluations from our students to see what works and what does not. It asks us to focus less on what we are teaching, and more on what our students are learning.

Implications

A number of interesting corollaries, elaborations, and implications that are relevant for understanding physics teaching come from Principle 1. The first is the realization that what we want our students to get is not simply "the content" but to build their understanding of that content into an accurate and effective mental model.

Corollary 1.1: The goal of physics teaching is to have students build the proper mental models for doing physics.

This helps us identify the point of teaching physics. I really want my students to do three things:

(a) develop the ability to reason qualitatively about physical processes;
(b) structure that content into coherent and appropriately organized—and appropriately accessible—mental models;
(c) learn how to apply that model to "do" physics in an expert and creative way.

These goals suggest that we should broaden our evaluation procedures. We traditionally test only the content and part of the student's skill in doing physics, usually (at least at the introductory level), in preset limited contexts.

Some of the characteristics of mental models clarify what is happening when students make mistakes. Often in listening to my students explain what they think I used to become confused and sometimes irritated. How can they say x when they know the contradictory principle y? How come they cannot get started on a problem when they certainly know the relevant principle? They just told it to me two minutes ago! How come they brought up that particular principle now? It does not apply here! The well-documented characteristics of mental models[22] listed above help me understand that these sorts of errors are natural and to be expected.

Corollary 1.2: It is not sufficient for students to "know" the relevant correct statements of physics. They also have to be able to gain access to them at the appropriate times; and they have to have methods of cross checking and evaluating to be certain that the result they have called up is truly relevant.

We must also test the underlying mental models that the students are developing. Traditional testing fails to do this, because many schemas can produce the correct solution to a problem. Even if the student goes through the same steps as we do, there is no guarantee that their schema for choosing the steps is the same as ours.[23] I once asked a student (who had done a problem correctly) to explain his solution. He replied: "Well, we have used all of the other formulas at the end of the chapter except this one, and the unknown starts with the same letter as is in that formula, so that must be the one to use."

Part of the way we have fooled ourselves with our current testing is that we are interested "in what students know." If they do not access the right "information" in an exam, we give them clues and hints in the wording to trigger access. But since an essential component of a mental model is the process for access to information, we are not testing the complete mental model. The student "has" the information, but it is inert and cannot be used or recalled except in very narrow almost preprogrammed situations.

To find out what our students really know we have to give them the opportunity to explain what they are thinking in words. We must also only give exam credit for reasoning, and not give partial credit when a student tries to hit the target with a blast of shotgun pellets and accidentally has a correct and relevant equation among a mass of irrelevancies. To know whether our students access the information in appropriate circumstances we have to give them more realistic problems—ones that relate directly to their real world experience and do not provide too many "physics clues" that specify an access path for them.

My next corollary relates to the problem that students often seem to listen to us but not to hear what we think we are trying to say.

Corollary 1.3: The student is not a tabula rasa (blank slate). Each one comes to us having had experiences with the physical world and having organized these experiences into mental models.

As physics teachers, we must realize that students come to us with naive mental models of how the physical world works. These are often referred to in the literature on physics education as preconceptions (or misconceptions).[24] Even experienced teachers can be surprised by this. A few years ago, after reading the ground-breaking articles of Halloun and Hestenes[3] in this journal on students' preconceptions in mechanics, I excitedly related a brief description of the results to one of my colleagues—someone whom I know to be a concerned teacher and a superb lecturer. He was skeptical at the idea that students had trouble learning Newton's first law because they had pre-existing mental models that were friction dominated. He insisted: "Just don't tell them about friction. They won't know about it if you don't tell them." This natural expectation of this experienced teacher is now strongly contradicted by an impressive body of data.

The presence of "false" preconceptions really is not so surprising if we think about our students' previous experience. Why should we be surprised that students think that any moving object will eventually come to a stop? In their direct personal experience that is always the case. It is even the case in the demonstrations we show in class to demonstrate the opposite! When I slide a dry-ice levitated puck across the lecture table, I catch it and stop it at the end of the table. If I did not, it would slide off the table, bounce, roll a short distance, and stop. Every student knows that. Yet I ask them to focus on a small piece of the demonstration—the stretch of about four or five seconds when the puck is sliding along the table smoothly—and extend that observation in their minds to infinity. The student and the teacher are focusing on different aspects of the same physical phenomena.[25]

Many teachers show surprise in response to the excellent educational physics studies that have graced the pages of this journal demonstrating that students regularly generalize their naive schemas incorrectly. Why should it be surprising that students think cutting off part of a lens will result in their seeing only part of an image?[26] Try it with a magnifying glass! (Yes, I know that is not a real image.)

Where do students get the idea that electricity is something that flows out of the wall and is used up in the object?[27] Why don't they think circuits? Although we do not always think about it, most of our students have had extensive experience with electricity by the time they arrive in our classes. When I said the current had to come in one wire and go out the other, one of my students complained: "If all the electricity goes back into the wall, what are we paying for?"

Corollary 1.4: Mental models must be built. People learn better by doing than by watching something being done.

This is sometimes expressed in the phrase: active learning works better than passive learning.[28] In most cases, this means that reading textbooks and listening to lectures is a poor way of learning.[29] This should not be taken as universally true! As physics teachers, most of us have had the experience of having a few "good" students in our lectures—students for whom listening to a lecture is an active process—a dialog between themselves and the teacher. Indeed, many of us have been that good student and remember lectures (at least some of them) as significant parts of our learning experience.[30] A similar statement can be made about texts. I remember with pleasure working through texts and lecture notes, reorganizing the material, filling in steps, and posing questions for myself to answer. Yet few of my students seem to know how to do this or even to know that this is what I expect them to do. This leads us to think about a fifth corollary.

Corollary 1.5: Many of our students do not have appropriate mental models for what it means to learn physics.

This is a "meta" issue. People build mental models not only for content, but also for how to learn and what actions are appropriate under what circumstances. Most of our students do not know what you and I mean by "doing" science or what we expect them to do. Unfortunately, the most common mental model for learning science in my classes seems to be:

(a) Write down every equation or law the teacher puts on the board that is also in the book.
(b) Memorize these, together with the list of formulas at the end of each chapter.
(c) Do enough homework and end-of-the-chapter problems to recognize which formula is to be applied to which problem.
(d) Pass the exam by selecting the correct formulas for the problems on the exam.
(e) Erase all information from your brain after the exam to make room for the next set of material.

I used to be flabbergasted to discover that when I stopped a lecture and said: "OK, here is a really important general principle for how to do things. It is not in the book but it comes from my years of experience as a physicist," my students would not write it down or consider it important, even if I wrote it on the board! (Well, after all, it was not going to be on the exam.)

I call the bulleted list above "the dead leaves model." It is as if physics were a collection of equations on fallen leaves. One might hold $s=1/2gt^2$, another $F=ma$, and a third $F=-kx$. These are each considered as of equivalent weight, importance, and structure. The only thing one needs to do when solving a problem is to flip through one's collection of leaves until one finds the appropriate equation. I would much prefer to have my students see physics as a living tree!

I like the term *mental ecology* to describe the mental model that tells students what mental model to apply in what set of circumstances. It is a more important goal to reshape our students' mental ecologies so that they expand their idea of learning to make it more constructive, take it out of the classroom into their everyday lives, and understand what science is and how to apply it, than it is to teach them to parrot back equations or solutions to turn-the-crank problems.

One final observation on the first principle is the following:

Constructing our own lectures and teaching materials can prove very useful in producing learning—in the teacher!

Haven't we all remarked: I only really understood E&M (or classical mechanics, or thermodynamics, or whatever) when I finally taught it? This is really dangerous! For those of us who love learning, the experience of lecturing and teaching is such a powerful learning experience that we do not want to give it up, even when it proves less effective for our students than other methods.

III. BUILDING ON A MENTAL MODEL: THE ASSIMILATION PRINCIPLE

The second and third principles have to do with the dynamics of modifying and extending one's mental models.

Principle 2: It is reasonably easy to learn something that matches or extends an existing mental model.

This principle states that mental models are not only the way that we organize our interactions with the world, but they also control how we incorporate new information and experiences.[31] (The question of how they are first established in young children is interesting—and controversial—but it does not really concern us here.) I use the term "assimilate" to emphasize adding something smoothly to an existing set.[32]

I pose three restatements and elaborations of this principle as corollaries to show what it means for teaching.

Corollary 2.1: It is hard to learn something we do not almost already know.

All students have things they know (some of which may be wrong!), things they are a bit familiar with, and things they have no knowledge about at all. In the last area my daughter would say they are "clueless."

I like to look at this as an archery target. What they know is the bull's-eye—a compact black area; what they know a little about is a gray area surrounding the black; and the clueless region is a white "rest of the world." To teach them something, I do best to hit in the gray. A class full of students is a challenge because all of their gray areas are not the same. I want to hit as many of the grays as possible with each paint-tipped shaft of information to turn gray areas black.

In communication studies, an important implication of this corollary is called the "given-new principle."[33] It states that new information should always be presented in a context that is familiar to the reader and that the context should be established first. The analogous statement is very important in physics teaching, especially at the introductory level. As physicists with years of training and experience we have a great deal of "context" that our students do not possess. Often we are as fish in water, unaware of this context and that it is missing in our students.

There are a number of specifics that we can cite that are given-new problems. We often use terms that students are not familiar with—or use in a different sense than we do. As a part of their study in the way speakers of English build their meaning of the term "force," Lakoff and Johnson[34] classified the characteristics of common metaphors using the term. Among their list of 11 characteristics, eight involved the will or intent of an individual! But most of us are so familiar with the technical meaning of force that we are surprised to learn that a significant fraction of our introductory students do not believe that a table exerts a force on a book it is supporting.[1] Why doesn't the book fall through? The table is just "in the way."

The problem caused by the interpretation of common speech words for technical ones is not a simple one. I know that the terms "heat," and "temperature" are not really distinguished in common speech and are used interchangeably for the technical terms temperature (average energy per degree of freedom), "internal energy," and heat (flow of internal energy from one object to another). In one class, I stated this problem up front and warned my students that I would use the terms technically in the lecture. Part way through I stopped, realizing that I had used the word heat twice in a sentence—once in the technical sense, once in the common speech sense.[35] It is like using the same symbol to stand for two different meanings in a single equation. You can occasionally get away with it,[36] but it is not really a good idea!

Putting new material in context is only part of the story. Our students also have to see the new material as having a plausible structure in terms of structures they are familiar with. We can state this as another useful corollary.

Corollary 2.2: Much of our learning is done by analogy.

This strongly counters the image of the student as a *tabula rasa*. This and the previous corollary make what students know at each stage critical for what we can teach them. Students always construct their knowledge, but what they construct depends on how what we give them interacts with what they already have.

One implication of these results is that we should focus on building structures that are useful for our students' future learning. I state this as a third corollary.

Corollary 2.3: "Touchstone" problems and examples are very important.

By a touchstone problem, I mean one that the student will come back to over and over again in later training. Touchstone problems become the analogs on which they will build the more sophisticated elements of their mental models.

It becomes extremely important for students to develop a collection of a few critical things that they really understand well.[37] These become the "queen bees" for new swarms of understanding to be built around. I believe this is why some problems have immense persistence in the community. Inclined plane problems really are not very interesting, yet the occasional suggestions that they be done away with are always resisted vigorously. I think the resisters are expressing the (usually unarticulated) feeling that these are the critical touchstone problems for building students' understanding of vector analysis in the plane. Corollary 2.3 is one reason why we spend so much time studying the mass on a spring. It is not really of much interest in itself, but it serves as a touchstone problem for all kinds of harmonic oscillation from electrical circuits up to quantum field theory.

Looking at a curriculum from the point of view of the mental models we want students to develop, their preexisting mental models, and touchstone problems can help us analyze what is critical in the curriculum, which proposed modifications could be severely detrimental, and which might be of great benefit.

Combining this with the discussion of access and linking above leads us to focus on the presence of a framework or structure within a course. It suggests that building a course around a linked series of touchstone problems could be of considerable assistance in helping students understand the importance and relevance of each element. Such a structure is sometimes referred to as a *story line*.

IV. CHANGING AN EXISTING MENTAL MODEL: THE ACCOMMODATION PRINCIPLE

Unfortunately, if students are *not* blank slates, sometimes what is written is—if not wrong—inappropriate for future learning in physics. Then it can seem as if we have run into a brick wall. This brings us to the next principle. I call this the "accommodation principle" to emphasize that changes have to be made in an existing structure. (Again, the term goes back to Piaget.)

Principle 3: It is very difficult to change an established mental model substantially.

Traditionally, we have relied on an oversimplified view of Principle 1, the patterning principle, to say: "Just let students do enough problems and they will get the idea eventually." Unfortunately, the principle does not apply in this form if they already have a mental model about the subject.

It has been demonstrated over and over again that simply telling somebody something does not easily change their deep ideas. Rather, what happens is that a poorly linked element is added with a rule for using it only in physics problems or for tests in one particular class. This and the fact that a mental model can contain contradictory elements is the reason why "giving more problems" can be ineffective. Once students learn how to do problems of a particular type, many will learn nothing more from doing more of them: new problems are done automatically without thinking. This also means that testing by varying homework problems slightly may be inadequate to probe the student's mental models of physics. More challenging tests involving a variety of modalities (problem solving, writing, interpreting, organizing) are required.

A few years ago I learned a lovely anecdote illustrating the barriers one encounters in trying to change a well-established mental model. A college physics teacher asked a class of beginning students whether heavy objects fall faster than light ones or whether they fall at the same rate. One student waved her hand saying "I know, I know." When called on to explain she said: "Heavy objects fall faster than light ones. We know this because Galileo dropped a penny and a feather from the top of the leaning tower of Pisa and the penny hit first." This is a touchstone example for me. It shows clearly that the student had been told—and had listened to—both the Galileo and the Newton stories. But she had transformed them both to agree with her existing mental model.[38]

Principle 3 can cause problems, both in getting students to change their mental models, and in getting ourselves to change the preconceptions we have about how students think! Fortunately, "difficult" does not mean "impossible." We have mechanisms that permit us to measure our mental models against the world and change them when we are wrong.[39] It appears as if the mechanism critically involves prediction and observation. The prediction must be made by the individual and the observation must be a clear and compelling contradiction to the existing mental model. A simple contradiction is not sufficient.

Posner *et al.*[40] suggest that changing an existing mental model requires that the change have the following characteristics (which I state as a corollary).

Corollary 3.1: In order to change an existing mental model the proposed replacement must have the following characteristics:

(a) It must be understandable.
(b) It must be plausible.
(c) There must be a strong conflict with predictions based on the existing model.
(d) The new model must be seen as useful.

The clearer the prediction and the stronger the conflict, the better the effect. A nice example of how this process works concerns physics teachers rather than their students. It explains why the response to the Halloun–Hestenes test has been so great. Many teachers of introductory physics have a mental model that says the teacher is successful in teaching the concepts if the students can average 75% on a traditional exam. These teachers look at the HH test and predict: "My students will be able to do very well on those problems. They are easy." Then their predictions fail miserably. On some critical questions, students average 20% or less and the conflict with the teacher's existing mental model is very strong. Many of the teachers who have gone through this process appear to be converted to a new way of looking at their students and what they know.[41]

Attempts are being made to combine the assimilation and the accommodation principles to yield new, more effective methods of teaching. John Clement[42] has proposed finding a series of *bridges or interpolating steps* that would help a student transform his or her mental model to match the accepted scientific one.

V. THE INDIVIDUALITY PRINCIPLE

One might be tempted to say: Fine. Let us figure out what the students know and provide them with a learning environment—lectures, demonstrations, labs, and problems—that takes them from where they are to where we want them to be. Since we all know that a few students get there from here using our current procedures, how come it does not work for all of them? We do in fact now know that the right environment can produce substantially better physics learning in most of the students taking introductory university physics.[43] But my final principle is a word of warning that suggests we should not be looking for a "magic bullet."

Principle 4: Since each individual constructs his or her own mental ecology, different students have different mental models for physical phenomena and different mental models for learning.

I like to call this the individuality or "linewidth" principle. This reminds us that many variables in human behavior have a large natural linewidth. The large standard deviation obtained in many educational experiments is not experimental error, it is part of the measured result! As scientists, we should be accustomed to such data. We just are not used to its being so broad and having so many variables. An "average" approach will miss everyone because no student is average in all ways.[44]

Implications

One implication of this is that different students can have different reasons for giving the same answer. If we formulate our questions too narrowly we may misinterpret the feedback we are getting. This observation has influenced the style of educational physics research in a way that at first seems strange to a physical scientist.

When we try to take our "first look" at what students are doing, it is very important to consider them one at a time and to interview them in depth, giving them substantial opportunity for "thinking aloud" and not giving them any guidance at all.

This approach is characteristic of many important educational studies. Instead of hoping to "average out" the variation by doing large statistical experiments, one focuses on it and tries to learn the range of possible approaches that students are taking. Of course, at a later stage, one wants to be able to interrogate large numbers of students in order to obtain the frequency with which various modes of thinking are occurring in the population at large. But many valuable studies in educational physics (and in cognitive studies in general) are done with a sample that seems very small to a physicist. An excellent example is the work of McDermott and Goldberg (Ref. 26). They start with extensive interviews of a fairly small number of students, then develop short answer exams based on those observations that can be applied to large groups, and finally test that those exams are giving the same results as the interviews. The various Hestenes tests were developed in a similar fashion.[45]

In addition to the fact that students have different experiences and have drawn different conclusions from them, their methods of approach may differ significantly. I state this as a corollary.

Corollary 4.1: People have different styles of learning.

There is by now a vast literature on how people approach learning differently. Many variables have been identified on which distributions have been measured. These include authoritarian/independent, abstract/concrete, and algebraic/geometric to name a few.[46] The first means that some students want to be told, others to figure things out for themselves. The second means that some students like to work from the general to the specific, some the other way round. The third means that some students prefer to manipulate algebraic expressions while others prefer to see pictures. Many of us who have introduced the computer in physics teaching have noted that some students want to be guided step by step, others try everything. These are only a few of the variables.

Once we begin to observe and use these differences in our students, we have to be exceedingly careful about how we use them. A preference does not mean a total lack of capability. Students who prefer examples with concrete numbers to abstract mathematical expressions may be responding to a lack of familiarity with algebra rather than a lack of innate ability. Many of our students preferences come from years of being rewarded for some activities (such as being good memorizers) and chastised for others (such as asking questions the teacher could not answer). Expanding our students' horizons and teaching them how to think sometimes requires us to overcome years of negative training and what they themselves have come to believe are their own preferences and limitations!

An interesting observation that has been made by a number of observers,[47] is that physics as a discipline requires learners to employ a variety of modalities (methods of understanding) and to translate from one to the other—words, tables of numbers, graphs, equations, diagrams, maps. Physics requires the ability to use algebra and geometry and to go from the specific to the general and back. This makes learning physics particularly difficult for many students. One of our goals should be to have our students understand this, th able to identify their own strengths and weaknesses, and while building on the former, strengthen the latter.

An important implication is the following.

Corollary 4.2: There is no unique answer to the question: What is the best way to teach a particular subject?

Different students will respond positively to different approaches. If we want to adopt the view that we want to teach all our students (or at least as many as possible), then we must use a mix of approaches and be prepared that some of them will not work for some students. An important set of studies that are just beginning to be done ask the question: What is the distribution function of learning characteristics that our students have in particular classes?

Another implication that is very difficult to keep in mind is:

Corollary 4.3: Our own personal experiences may be a very poor guide for telling us what to do for our students.

Physics teachers are an atypical group. We selected ourselves at an early stage in our careers because we liked physics for one reason or another. This already selects a fairly small subclass of learning styles from the overall panoply of possibilities. We are then trained for approximately a dozen years before we start teaching our own classes. This training stretches us even further from the style of approach of the "typical" student. Is it any wonder why we do not understand most of our beginning students and they do not understand us? I will never forget one day a few years ago when a student in my algebra-based introductory physics class came in to ask about some motion problems. I said: "All right, let us get down to absolute basics. Let's draw a graph." The student's face fell, and I realized suddenly that a graph was not going to help him at all. I also realized that it was going to be hard for me to think without a graph and to understand what was going through the student's mind. I never minded doing without derivatives—motion after all is the study of experimental calculus and you have to explain the concept (maybe without using the word) even in a non-calculus-based class; but I have found it difficult to empathize with students who come to physics and cannot read a graph or reason proportionately.[1] It takes a special effort for me to figure out the right approach.

This is very natural given the earlier principles. Our own personal mental models for how to learn come from our own experiences. However, to reach more of our students than the ones who resemble ourselves, we will have to do our best to get beyond this. It makes the following principle essential.

Corollary 4.4: The information about the state of our students knowledge is contained within them. If we want to know what they know, we not only have to ask them, we have to listen to them!

VI. CONCLUSION

The typical university course is a complex structure. It involves physics content, a teacher, perhaps graders or teaching assistants, a classroom, a laboratory, and, for each class, a particular set of students. Above all, it involves expectations and contexts for both the teacher and the students. If we are to make serious progress in reaching a larger fraction of our students, we will have to shift our emphasis from the physics content we enjoy and love so well to the students themselves and their learning. We must ask not only what do we want them to learn, but what do they know when they come in and how do they interact with and respond to the learning environment and content we provide.

The principles we are learning from cognitive studies can provide a framework for how we think about the complex issues of teaching and learning. The four principles that I have presented can help us begin to construct such a framework.

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[1] A. Arons, *A Guide to Introductory Physics Teaching* (Wiley, New York, 1990).

[2] D. E. Trowbridge and L. C. McDermott, "Investigation of student understanding of the concept of velocity in one dimension," Am. J. Phys. **48**, 1020–1028 (1980) "Investigation of student understanding of the concept of acceleration in one dimension," *ibid.* **49**, 242–253 (1981).

[3] A. Halloun and D. Hestenes, "The initial knowledge state of college physics students," Am. J. Phys. **53**, 1043–1055 (1985); "Modeling instruction in mechanics," *ibid.* **55**, 455–462 (1987).

[4] R. K. Thornton and D. R. Sokoloff, "Learning motion concepts using real-time microcomputer-based laboratory tools," Am. J. Phys. **58**, 858–867 (1990).

[5] L. C. McDermott, "Millikan Lecture 1990: What we teach and what is learned—Closing the gap," Am. J. Phys. **59**, 301–315 (1991); "Guest Comment: How we teach and how students learn—A mismatch?" *ibid.* **61**, 295–298 (1993), and references therein.

[6] A. van Heuvelen, Bull. Am. Phys. Soc. **36**, 1359 (April, 1991).

[7] One that has been very useful for me is the article by D. Hestenes, "Toward a modeling theory of physics instruction," Am. J. Phys. **55**, 440–454 (1987).

[8] I use the term "cognitive studies" as opposed to the term "cognitive science," which is more prevalent in the literature, advisedly. The field is extremely broad and cuts across many disciplines. Very little in cognitive science satisfies the scientific criteria we are used to in physics of being precisely stated and well-tested experimentally, as well as useful. So as not to raise the expectations of my scientist readers as to the nature of the information precised here, I use the weaker term (and use it as a singular noun like "physics").

[9] There are a few notable exceptions. Piaget has made extensive studies of how children build their concepts of the physical world, and a number of trained physicists such as Fred Reif, John Clement, and Jose Mestre, among others, could be counted as cognitive scholars.

[10] H. Gardner, *The Mind's New Science: A History of the Cognitive Revolution* (Basic Books, New York, 1987).

[11] *Mental Models*, edited by D. Gentner and A. L. Stevens (Lawrence Erlbaum Associates, Hillsdale, NJ, 1983).

[12] A. Collins and E. E. Smith, *Readings in Cognitive Science* (Morgan Kauffmann, San Mateo, CA, 1988).

[13] B. Inhelder and J. Piaget, *The Growth of Logical Thinking from Childhood to Adolescence* (Basic Books, New York, 1958).

[14] L. B. Resnick, "Mathematics and science learning: A new conception," Science **220**, 447–478 (1983); S. Carey, "Cognitive science and science education," Am. Psych. **41**, 1123–1130 (1986); R. Driver, "Students' conceptions of the learning of science," Int. J. Sci. Ed. **11**, 481–490 (1989).

[15] D. Norman, *The Design of Everyday Things* (Basic Books, New York,

1990). (This book was originally published under the title: *The Psychology of Everyday Things*.)

[16]L. McDermott, "Research on conceptual understanding in mechanics," Phys. Today 37(7), 24–32 (1984); J. Mestre and J. Touger, "Cognitive research—what's in it for physics teachers?," The Phys. Teach. 27, 447–456 (September, 1989); R. Fuller, "Solving physics problems—how do we do it?," Phys. Today 35 (9), 43–47 (1982).

[17]D. H. Hubel, "The visual cortex of the brain," Sci. Am. 225 (11), 54 (1963); "The brain," *ibid.* **241**: (3), 44 (1979).

[18]The page is blank with two marks: a black dot and a cross about six inches apart. One closes one eye and focuses on the black dot with the other. The page is oriented so the cross is off to one side. By moving one's head back and forth, one finds a distance at which the cross appears to vanish.

[19]One must take particular care with these terms in the cognitive literature as they are used by different authors in different senses.

[20]This list of properties is based in part on D. Norman, "Some observations on mental models," in Ref. 11, pp. 7–14, and references therein.

[21]Actually, a very wide variety of educators and cognitive specialists consider themselves constructivists. The statement here is not really extreme in the current spectrum of views. Radical constructivists reject the idea that knowledge has any base outside of construction in the human mind, a view which leads to results that could be classified as solipsism.

[22]Andy diSessa has documented and attempted to classify some of the levels of principles that people create and their modes of access to them. Although his papers are rather technical, he uses many examples from physics and includes references to the large literature on the subject. See for example, "Phenomenology and the evolution of intuition," in Ref. 11, pp. 15–33; or "Toward an epistemology of physics," Cognit. Instruct. **10**, issues 2 and 3, 105–226 (1993).

[23]The difficulty is that the mapping from underlying schema to problem solving steps is not one-to-one. As nice example of this is given in a recent article in this journal: J. Bowden *et al.*, "Displacement, velocity, and frames of reference: phenomenographic studies of students' understanding and some implications for teaching and assessment," Am. J. Phys. **60**, 262–269 (1992).

[24]I prefer the term preconceptions or naive schemas, with "naive" used in the nonpejorative sense of "untutored." The typical student's naive schema about motion and mechanics, for example, is not wrong. It correctly correlates their observations at low velocities in friction-dominated systems. It just does not provide them with the power and range of the schema we are trying to substitute for it. See Refs. 1–5 and the references referred to therein for details and documentation.

[25]This argument is made in a slightly different context in T. S. Kuhn, *The Structure of Scientific Revolutions*, 2nd ed. (Chicago University Press, Chicago, 1970).

[26]F. C. Goldberg and L. C. McDermott, "An investigation of student understanding of the real image formed by a converging lens or concave mirror," Am. J. Phys. 55, 108–119 (1987); "An investigation of student understanding of the images formed by plane mirrors," Phys. Teach. 34, 472–480 (1986).

[27]R. Cohen, B. Eylon, and U. Ganiel, "Potential difference and current in simple electric circuits: A study of students' concepts," Am. J. Phys. **51**, 407–412 (1983).

[28]We have to be careful what we mean by "active learning." This shouldn't necessarily be taken to mean that "solving more problems is the answer." See the discussion after principle 3.

[29]J. S. Brown, A. Collins, and P. Duguid, "Situated cognition and the culture of learning," Educational Researcher **18** (1), 32–42 (Jan–Feb 1989), and references therein.

[30]In many research groups, a seminar more resembles a panel discussion than a lecture. These are very active learning experiences, both for the speaker and the listener.

[31]J. D. Bransford and M. K. Johnson, "Contextual prerequisites for understanding: Some investigations of comprehension and recall," J. Verbal Learning Verbal Behavior **11**, 717–726 (1972); "Considerations of some problems of comprehension," in *Visual Information Processing* (Academic, New York, 1973).

[32]The term "assimilation" and the matching term "accommodation" used in the next section were introduced by J. Piaget.

[33]H. Clark and S. Haviland, "Comprehension and the given-new contract," in *Discourse Production and Comprehension*, edited by R. Freedle (Lawrence Erlbaum, Hillsdale, NJ, 1975).

[34]G. Lakoff and M. Johnson, *Metaphors We Live By* (University of Chicago Press, Chicago, 1980), p. 70.

[35]"If there is no heat flow permitted to the object we can still heat it up by doing work on it."

[36]The energy levels of hydrogen in a magnetic field could be written as $E_{nlm}=E_n-[e\hbar/(2m)]mB$ where the m in the denominator is the electron mass and the one in the numerator is the z component of the angular momentum. Most physicists can correctly interpret this abomination without difficulty.

[37]In addition to giving them centers on which to build future learning, knowing a few things well gives the student a model of what it means to understand something in physics. This valuable point that has been frequently stressed by Arnold Arons. It is an essential element in the mental ecology of a scientist.

[38]Of course the students' mental model in this case is in fact correct. Lighter objects do fall more slowly than heavy ones if they fall in air, and few of us have much direct experience with objects falling in a vacuum. But that observation does not yield a useful idealization. The observation that objects of very different mass fall in very nearly the same way does.

[39]Unfortunately, these mechanisms appear to atrophy if unused.

[40]G. J. Posner, K. A. Strike, P. W. Hewson, and W. A. Gertzog, "Accommodation of a scientific conception: Toward a theory of conceptual change," Sci. Ed. **66** (2), 211–227 (1982).

[41]A nice description of this process is given in Eric Mazur's brief but insightful article on teaching, "Qualitative vs quantitative thinking: Are we teaching the right thing?," Opt. Phot. News 3, 38 (February, 1992).

[42]J. Clement, "Overcoming students' misconceptions in physics: The role of anchoring intuitions and analogical validity," *Proceedings of the Second International Seminar: Misconceptions and Educational Strategies in Science and Mathematics III*, edited by J. Novak (Cornell University, Ithaca, NY, 1987); "Not all preconceptions are misconceptions: Finding 'anchoring conceptions for grounding instruction on students' intuitions," Int. J. Sci. Ed. **11** (special issue), 554–565 (1989).

[43]P. Laws, "Calculus-based physics without lectures," Phys. Today **44** (12), 24–31 (December, 1991); A. Van Heuvelen, "Overview, case study physics," Am. J. Phys. **59**, 898–907 (1991); R. Hake, "Socratic pedagogy in the introductory physics laboratory," Phys. Teach. **33**, (1992); L. C. McDermott and P. S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity. Part I: Investigation of student understanding," Am. J. Phys. **60**, 994–1003 (1992); *ibid.* **61**, 81(E) (1993); P. Shaffer and L. C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity. Part II: Design of an instructional strategy," *ibid.* **60**, 1003–1013 (1992).

[44]This is analogous to the story of the three statisticians who went hunting deer with bows and arrows. They came across a large stag and the first statistician shot—and missed to the left. The second statistician shot and missed to the right. The third statistician jumped up and down shouting "We got him!"

[45]See Ref. 3; D. Hestenes, M. Wells, and G. Swackhammer, "Force concept inventory," Phys. Teach. **30** (3), 141–158 (1992); D. Hestenes and M. Wells, "A mechanics baseline test," *ibid.* **30** (3), 159–166 (1992).

[46]D. A. Kolb, *Experiential Learning: Experience as a Source of Learning and Development* (Prentice-Hall, Englewood Cliffs, 1984); N. Entwistle, *Styles of Integrated Learning and Teaching: An Integrated Outline of Engelwood Cliffs Educational Psychology for Students, Teachers, and Lecturers* (Wiley, New York, 1981); H. Gardner, *Frames of Mind* (Basic Books, 1983).

[47]P. Laws and D. Hestenes (private communication).