Implications of cognitive studies for teaching physics
Edward F. Redish
Department of Physics, University of Maryland, College Park, Maryland 20742-4111
(Received 20 March 1992, accepted 13 April 1994)

I. INTRODUCTION

Many of us who have taught introductory physics for many years recall with dismay a number of negative experiences: a graph but cannot say what it means; a top student who can solve all the problems but not give an overview or simple derivation; many students of varying abilities who memorize without understanding despite our most carefully crafted and elegant lectures.

As physics teachers who care about physics, we have a tendency to focus on the physics content we are teaching. We often are concerned for those students who are like we were—that small fraction of our students who find physics interesting and exciting and who will be the next generation of professional physicists. But the changes in our society and in the role of technology for the general public mean that we must change the way we are teaching. It no longer suffices to reproduce ourselves. Society has a great need not only for a few technically trained people, but for a large group of individuals who understand science.

During the past decade, data have built up that demonstrate that as physics teachers we fail to make an impact on the way a majority of our students think about the world.1-3 We have redrafted our testing so that the students can succeed and we have then either fooled ourselves or we are teaching them successfully or lowered our standards by eliminating understanding from our definition of successful learning. Alan van Heuvelen3 has remarked that in his study of a typical introductory lecture class, 20% of the students entered the first semester of an introductory calculus-based physics class as Newtonian thinkers. The impact of the course was to increase that number to 25%. If we want to reach all of our students, we must pay more attention to how students learn and how they respond to our teaching. We must treat the teaching of physics as a scientific process.

A few physicists have begun to perform detailed experiments to determine what our students are thinking and what works in the teaching of physics. Some of their articles are of tremendous importance and I believe should be read by every physics teacher (see Refs. 4 and 5, and references therein). But even among these few articles, only a small fraction of the authors attempt to place their results in a general theoretical framework—to give us a way of thinking about and organizing the results. Those of us in physics know well that advancements in science is a continual dance between the partners of theory and experiment, first one leading, then the other. It is not sufficient to collect data into a "wizard's book" of everything that happens. That's not science. Neither is it science to quote high-blown theories untainted by "reality checks." Science must build a coherent and clear picture of what is happening at the same time as it continually confirms and refutes that picture against the real world.

The time has come for us to begin the development of a framework for understanding and talking about student learning. Some of the results of the past few decades in cognitive studies4 begin to provide such a framework.

Cognitive studies focus on how people understand and learn. It is still an amorphous field, and it is not yet really a single discipline. It overlaps many areas from anthropology to neuropsychology. It may not yet be "a science" as we in physics use the term, but developments in the past few decades have changed drastically what we know about how the mind works.

The issue of how to teach physics is a difficult one: the attempt of a naive student to build a good understanding of physics involves many intricate processes over a long period of time. These processes tend to be much more complex than those most cognitive scholars have addressed.5 Nonetheless, some of the basic ideas of cognitive studies appear to be both firmly grounded and useful to the teacher of physics.

In this essay I briefly review some of the lessons I have learned from cognitive studies. This is not a review article, but a narrow selection from a small slice of a large field. For those interested in a more substantial introduction to cognitive studies I recommend Howard Gardner's historical overview,6 the collection of articles assembled by Gentner and Stevens,7 and some of the articles in the spring volume collected by Collins and Smith.12 These will give an entry point into the modern cognitive literature. The book by Inhelder and Piaget13 has lots of discussion of experiments on how adolescents learn physics. Some articles by leading educational specialists also can help link to the existing literature.14 Just for fun, I have to add Donald Norman's delightful book on how people interact with objects in their world.15 For an introduction to the physics education research literature, Arnold Aron's book17 and a few review articles18 provide an appropriate entry point.

I have grouped what I have learned from the cognitivists into four broad principles with elaborative corollaries. One of the things students of cognitive processes have learned about thinking is that it is fuzzy. The sharp, crisp operations of formal logic or the digital computer are inappropriate models for the way most people think. Therefore, it is not correct to call these principles "theorems" or "laws of cognitive science." Nor is it correct to use them as such. They cannot provide us with hard and fast rules for what to do. Using them ineptly without reference to experimental data can lead us to the wrong conclusions. But I have found that they help me to organize my thinking about my students and to refocus my attention. Instead of concentrating only on the organization of the physics content, I now also pay attention to what my students are doing when they interact with a physics course. This is not to suggest that an emphasis on content is somehow unimportant or should be neglected. What we are teaching is important, but it must be viewed in the context of what our students learn.
II. BUILDING PATTERNS: THE CONSTRUCTION PRINCIPLE

The fundamental change that has led to the breakthroughs in cognitive studies is the past few decades has been a new willingness to model what is happening in the mind in terms of inferred structures. For the first half of this century,14 studies of thinking were severely constrained by the “behaviorist” philosophy that one should formulate all one's theories out in terms of direct observations. This is like the S-matrix theory of elementary particles which insisted that we should only formulate our theories in terms of observable particles and their scattering amplitudes. Elementary particle physicists only made their breakthrough when they were willing to formulate their ideas in terms of quarks and gluons—particles which could only be inferred and not be seen directly. Cognitive scholars started to make real progress when they began to be willing to formulate how people were thinking in terms of mental patterns or models that could not be directly observed or measured.


Principle 1: (Weak form) People tend to form mental pat-
terns. This is a fundamental hypothesis about how the mind works. On some levels, there is direct observation of this mental processing by patterning. For example, it has been demonstrated in detail that we process visual information or a variety of levels to form patterns beginning with the first layer of nerve cells attached to the retina, and the pro-
cess continues through many stages deep into the brain.15 I once attended a physics colloquium given by Jerome Letvin on the subject of blind spots in the visual field. He passed out the standard blind-spot demonstration pages16 that let us clearly find the blind spot in our eye. We then moved the end of a pencil into our blind spots and saw the end of the pencil disappear as it bit hit. Yet there was a “blank spot” in the visual field. The brain fills in the background—here the same simple white of a blank page. But is will fill in even a quite complex pattern. If the page had been covered with fluid, my brain would still have filled in my blind spot with the appro-
priate pattern. But note that the patterning was not suffi-
ciently sophisticated to produce the “right” answer! My au-
tomatic filling led me see a pencil in the blind spot, not the
rest of the pencil.

The tendency of the human mind to form patterns is not just limited to the analysis of sensory data. This leads me to state the principle in a stronger (and more relevant) form.

Principle 1: (Strong form) People tend to organize their
experiences and observations into patterns or mental mod-
els.

I use the term mental model for the collection of mental patterns people build to organize their experiences related to a particular topic. I use the term schema (pl. schemata or schemata) to describe the basic elements of these mental models. I think of a schema as a “chunk” or “object” (in the sense of object-oriented programming). It is a collection of closely linked data together with some processes and rules for use. He is careful of the use of the word “model.” It tends to convey something clockwork—a mechanism that has links and rules and operates in a well-defined way. These are not the characteristics of our mental models.

The characteristics of mental models and schemas are still vigorously debated.19 Despite attempts to build a general representational system for mental models, none has yet been widely accepted. However, some results are clear.20


Properties of mental models:

- Mental models have the following properties:
  1. They consist of propositions, images, rules of procedure, and statements as to when and how they are to be used.
  2. They may contain contradictory elements.
  3. They may have many levels of structure.
  4. People may not know how to “run” the procedures present in their mental models.
  5. Mental models are not rigid collections of specific activities—sometimes very time consuming and difficult—in order to avoid a little bit of serious thinking.

This inferred structuring of mental models is distinctly different from what we usually assume when teaching phys-
ics. We usually assume that our students either know some-
thing or they do not. The view of mental models we learn from cognitive scholars suggests otherwise. It suggests that students may hold contradictory elements in their minds without being aware that they contradict.

I had an interesting experience that illustrated me vividly the surprising fact that our mental models may contain contradictory elements. Ron Thornton visited the University of Maryland a few years ago to give a seminar on his now famous work on using the Sonic Ranger to teach the concept of velocity. 2 The Ranger detects the position of an object using sound and can display the position or the velocity of the detected object on a computer screen in live time. Thornton set up the Ranger to display velocity and had the computer show a preset pattern (a square wave). He then called me up to the front of the room to see as a guinea pig and try to walk so my velocity matched the preset pattern.

I had no hesitation in doing this. I had been teaching physics for nearly 20 years and felt perfectly comfortable with the concept of velocity. I did my first trial without thinking; I walked backward until my velocity reached the height of the preset square wave. Then I stopped and my velocity dropped to zero immediately! I asked for another chance, and this time, purporting my “speedability,” I was able to reduce the curve without difficulty.

What this experience said to me was that, for normal walking, I still maintained a naive (but appropriate!) position-dominated proposition is our mental model of mo-
tion. I also had a correct proposition for the concept of ve-
cocity, but I had to consciously apply a rule telling me to use it.

I have also had personal experiences illustrating character-
istic-6. I once spent an hour searching through my hard drive and piles of floppy disks to find a short paragraph (three sentences) that I needed again. When I found it, I realized it would have taken me only five minutes to have rewritten it from scratch. A nice example of this characteristic is Donald Norman's study of how people use complex calculators (Ref. 20).

An important aspect of Principle 1 is that "people... or-
ganize their experiences into mental models" with the em-
phasis on the fact that people must build their own mental models. This is the cornerstone of the educational philosophy known as constructivism. For this reason, I refer to Principle 1 as "The Construction Principle."

An extreme statement of constructivism is: 21

797

Edward F. Redish

797

You cannot teach anybody anything. All you can do as a teacher is to make it easier for your students to learn.

Of course, facilitation can be critical to the learning process. Students seem to benefit from a variety of teaching methods, including collaborative learning, discussion, and interactive activities. However, it is important to recognize that individual differences in learning style and pace play a significant role in the effectiveness of teaching.

Some students may benefit from visual aids, such as diagrams and videos, while others may prefer auditory explanations or hands-on activities. It is essential for teachers to be adaptable and flexible in their approach to teaching, recognizing that effective communication requires understanding the needs and preferences of each student.

Incorporating technology into the classroom can also enhance learning experiences. Digital tools and resources, such as online tutorials and interactive simulations, can provide students with opportunities to engage with course material in a more dynamic and interactive manner. Teachers should be encouraged to explore and integrate these tools into their teaching practices.

In conclusion, while there is no single "correct" way to teach, it is crucial for educators to be open-minded and willing to experiment with different strategies to find what works best for their students. By cultivating a learning environment that values diversity and encourages student autonomy, educators can help students develop critical thinking and problem-solving skills that will serve them well throughout their lives.

Edith F. Redish
think about it, most of our students have had extensive experience with electricity by the time they arrive in our classes. When I said the current had to come in one wire and go out the other, one of my students complained: "If all the electricity goes back into the wall, what are we paying for?"

Corollary 1.4: Mental models must be built. People learn better by doing than by watching something being done. This is sometimes expressed in the phrase: active learning works better than passive learning. In most cases, this means that reading textbooks and listening to lectures is a poor way of learning. This should not be taken at "universally true. As physics teachers, most of us have had the experience of having a few "good" students in our lectures—students for whom listening to a lecture is an active process—a dialogue between themselves and the teacher. Indeed, many of us have been that good student and remember lectures (at least some of them) as significant parts of our learning experience. A similar statement can be made about labs. I remember with pleasure working through tests and lab notes, organizing the material, filling in steps, and posing questions for myself to answer. Yet few of my students seem to know how to do this or even to know that this is what I expect them to do. This leads us to think about a fifth corollary.

Corollary 1.5: Many of our students do not have appropriate mental models for what is meant to learn physics. This is a "meta" issue. People build mental models not only for content, but also for how to learn and what actions are appropriate under what circumstances. Most of our students do not know what you and I mean by "doing" science or what we expect them to do. Unfortunately, the most common mental model for learning science in my classes seems to be:

(a) Write down every equation or law the teacher puts on the board that is also in the book.
(b) Memorize the end of each chapter problem with the list of formulators at the end of each chapter.
(c) Do enough homework and end-of-the-chapter problems to recognize which formula is to be applied to which problem.
(d) Pass the exam by selecting the correct formulas for the problems on the exam.
(e) Erase all information from your brain after the exam to make room for the next set of material.

I used to be flabbergasted to discover that when I stopped a lecture and said: "OK, here is a really important general principle for how to do things. It is not in the book but it comes from my years of experience as a physicist," my students would not write it down or consider it important, even if I wrote it on the board (Well, after all, it was not going to be on the exam.)

I call the bulleted list above "the dead leaves model." It is as if physics were a collection of equations on fallen leaves. One might hold $x = 1/2 gt^2$, another $F = ma$, and a third $F = -kx$. These are each considered as of equivalent weight, importance, and structure. The only thing one needs to do when solving a problem is to flip through one's collection of leaves until one finds the appropriate equation. I would much prefer to have my students see physics as a living tree.

I like the term mental ecology to describe the mental model that tells students what mental model to apply in what set of circumstances. It is a more important goal to reshape our students' mental ecologies so that they expand their idea of learning to make it more constructive, take it out of the classroom into their everyday lives, and understand what science is and how to apply it, than to teach them to parrot back equations or solutions to turn-the-crank problems. One final observation on the first principle is the following: Constructing our own leaures and teaching materials can prove very rewarding.

Haven't we all remarked: I only really understood E&M (or classical mechanics, or thermodynamics, or whatever) when I finally taught it. This is really dangerous! For those of us who love learning, the experience of lecturing and teaching is such a powerful learning experience that we do not seek a "crutch" for it. For, it proves less effective for our students than other methods.

III. BUILDING ON A MENTAL MODEL: THE ASSIMILATION PRINCIPLE

The second and third principles have to do with the dynamics of modifying and extending one's mental models.

Principle 2: It is reasonably easy to learn something that matches or extends an existing mental model. This principle states that mental models are not only the way that we organize our interactions with the world, but they also control how we incorporate new information and experiences. (The question of how they are first established in young children is interesting—and controversial—but it does not really concern us here.) I use the term "assimilate" to describe the way that something fits into an existing mental model. I pose three refinements and elaborations of this principle as corollaries to show what it means for teaching.

Corollary 2.1: It is hard to learn something we do not almost already know.

All students have things they know (some of which may be wrong), things they are a bit familiar with, and things they have no knowledge about at all. In the last area my daughter would say they are "clueless.

I like to look at this as an archery target. What they know is the bull's-eye—a compact black area; what they know a little about is a gray area surrounding the black; and the clueless region is a white "rest of the world." To teach them something, I do these to hit in the gray. A class full of students is a challenge because all of their gray areas are not the same. I want to hit as many of the grays as possible with each painted-tipped shaft of information to turn gray areas black.

In communication studies, an important implication of this corollary is called the "given-new principle." It states that new information should always be presented in a context that is familiar to the reader and that the context should be established first. The analogous statement is very important in physics teaching, especially at the introductory level. As physicists with years of training and experience we have a great deal of "context" that our students do not possess. Often we are as fish in water, unaware of this context and that it is missing in our students.

There are a number of specifics that we can cite that are given-new problems. We often use terms that are not familiar with—or use in a different sense than we do. As a part of their study in the way speakers of English build their meaning of the term "force," Lakoff and Johnson classified the characteristics of common metaphors using the term. Among their list of 11 characteristics, eight involved

990 Am. J. Phys. 62, No. 9, September 1994

Edward F. Fisch 999
the will or intent of an individual. But most of us are so familiar with the technical meaning of force that we are surprised to learn that a significant fraction of our introductory students do not believe that a table exerts a force on a book it is supporting. Why doesn't the book fall through the table is just "in the way." The problem caused by the interpretation of common speech words for technical ones is not a simple one. I know that the terms "heat," and "temperature" are not really distinguished in common speech and are used interchangeably for the technical terms temperature (average energy per degree of freedom), "internal energy," and heat (flow of internal energy from one object to another). If one class, I stated this problem up front and warned my students that I would use the terms technically in the lecture. Part way through I stopped, realizing that I had used the word heat twice in a sentence—one in the technical sense, once in the common speech sense. It is like using the same symbol to stand for two different meanings in a single equation. You can occasionally get away with it, but it is not really a good idea.

Putting new material as context is only part of the story. Our students also have to see the new material as having a plausible structure in terms of structures they are familiar with. We can state this as another useful corollary.

Corollary 2.2: Much of our learning is done by analogy. This strongly counters the image of the student as a tabula rasa. This and the previous corollary make what students know at each stage critical for what we can teach them. Students always construct their knowledge, but what they construct depends on how what we give them interacts with what they already have.

One implication of these results is that we should focus on building structures that are useful for our students' future learning. I state this as a third corollary.

Corollary 2.3: Touchstone problems and examples are very important. By a touchstone problem, I mean one that the student will come back to over and over again in later training. Touchstone problems become the analogs on which they will build the most sophisticated elements of their mental models. It becomes extremely important for students to develop a collection of a few critical things that they really understand well.17 These become the "queen bees" for new swarms of understanding to be built around. I believe this is why some problems have immense persistence in the community. Ini- nclined plane problems really are not very interesting, yet the occasional suggestions that they be done away with are always vigorously resisted. I think the resistors are expressing the (usually unarticulated) feeling that these are the critical touchstone problems for building students' understanding of vector analysis in the plane. Corollary 2.3 is one reason why we spend so much time studying the max on a spring. It is not really of much interest in itself, but it serves as a touchstone problem for all kinds of harmonic oscillation from electrical circuits up to quantum field theory.

Looking at a curriculum from the point of view of the mental models we want students to develop, their pre-existing mental models, and touchstone problems can help us analyze who is critical in the curriculum, which proposed modifications could be severity detrimental, and which are of great benefit.

Combining this with the discussion of access and linking above leads us to focus on the presence of a framework or structure within a course. It suggests that building a course around a linked series of touchstone problems could be of considerable assistance in helping students understand the importance and relevance of each element. Such a structure is sometimes referred to as a story line.

IV. CHANGING AN EXISTING MENTAL MODEL: THE ACCOMMODATION PRINCIPLE

Unfortunately, if students are not blank slates, sometimes what is written in—is not wrong—improper for future learning in physics. Then it can seem as if we have run into a brick wall. This brings us to the next principle. I call this the "accommodation principle" to emphasize that changes have to be made in an existing structure. (Again, the term goes back to Piaget.)

Principle 3: It is very difficult to change an established mental model substantially.

Traditionally, we have relied on an oversimplified view of Principle 1, the patterning principle, to say: "Just let students do enough problems and they will get the idea eventually." Unfortunately, the principle does not apply in this form if they already have a mental model about the subject.

It has been demonstrated over and over again that simply telling somebody something does not easily change their deep ideas. Rather, what happens is that a poorly linked element is added with a rule for using it only in physics problems or for tests in one particular class. This and the fact that a mental model can contain contradictory elements is the reason why "giving more problems" can be ineffective. Once students learn how to do problems of a particular type, many will learn nothing more from doing more of them: new problems are done automatically without thinking. This also means that testing by varying homework problems slightly may be inadequate to probe the student's mental models of physics. More challenging tests involving a variety of mo- dalities (problem solving, writing, interpreting, organizing) are required.

A few years ago I learned a lovely anecdote illustrating the barriers one encounters in trying to change a well-established mental model. A college physics teacher asked a class of beginning students whether heavy objects fall faster than light ones or whether they fall at the same rate. One student waved her hand saying "I know, I know." When called on to explain she said: "Heavy objects fall faster than light ones. We know this because Galileo dropped a penny and a feather from the top of the leaning tower of Pisa and the penny hit first." This is a touchscreen example for me. It shows clearly that the student had been told—and had listened to—but the Galileo story doesn't matter. But she had changed them both to agree with her existing mental model.

Principle 3 can cause problems, both in getting students to change their mental models, and in getting ourselves to change the preconceptions we have about how students think! Fortunately, "difficulty" does not mean "impossible." We have mechanisms that permit us to change our mental models against the world and change them when we are wrong.18 It appears as if the mechanism critically involves pre-existing mental models. The pre-existing mental models must be made public, the individual and the observation must be a clear and com- pelling contradiction to the existing mental model. A simple contradiction is not sufficient.

Posner et al.19 suggest that changing an existing mental model requires that the change have the following character-istics (which I state as a corollary).


Edward R. Redish 800
Corollary 3.1: In order to change an existing mental model the proposed replacement must have the following characteristics:

(a) It must be understandable.
(b) It must be plausible.
(c) There must be a strong conflict with predictions based on the existing model.
(d) The new model must be shown as useful.

The clearer the prediction and the stronger the conflict, the better the effect. A nice example of how this process works is the case of the physics teachers who use their subjects. It explains why the response to the Halloun-Hestenes test has been so great. Many teachers of introductory physics have a mental model that describes physics and the ideas behind some key concepts. If the students' test average 75% on a traditional exam. These teachers look at the HST test and predict: "My students will still be able to do very well on those problems. They are easy." Then their predictions fail miserably. Some critical questions, students average 20% or less and the conflict with the teachers' existing mental model is very strong. Many of the teachers who have already experienced this process appear to be converted to a new way of looking at their students and what they know. 4

Attempts are being made to combine the assimilation and the accommodation principles to yield new, more effective methods of teaching. John Conner 45 has proposed finding a series of bridging or interposing steps that would help a student transform his or her mental model to match the accepted scientific one.

V. THE INDIVIDUALITY PRINCIPLE

One might be tempted to say: Fine. Let us figure out who the students know and provide them with a learning environment—lectures, demonstrations, labs, and problems—that takes them from where they are to where we want them to be. Since we all know that a few students get from here using our current procedures, how come it does not work for all of them? No, we must ask now that if environments can produce substantial better physics learning in most of the students taking introductory university physics. 45 But my final principle is a word of warning: do not start spraying a "magic bullet."

Principle 4: Since each individual constructs his or her mental model, each student has different mental models for physical phenomena and different mental models for learning.

Like I said, this is the individuality or "line of sight" principle. This reminds us that many variables in human behavior have a large natural linewidth. The large standard deviation obtained in many educational experiments is not experimental error, it is part of the measured result. As scientists, we should be accustomed to such data. We just are not used to its being so broad and having so many variables. As "averaging" approach will miss everyone because no student's average is in all ways.

Implications

One implication of this is that different students may have different reasons for giving the same answer. If we formulate our questions too narrowly we may misinterpret the feedback we are getting. This observation has influenced the style of educational physics research in a way that at first seems strange to a physical scientist.


Edward F. Redish
The principles we are learning from cognitive studies can provide a framework for how we think about the complex issues of teaching and learning. The four principles that have presented can help us begin to construct such a framework.

ACKNOWLEDGMENTS

This article arises from the valuable discussions I have had with many friends and colleagues, especially Arnold Arons, Seth Chaklin, Dewey Djikstra, John Guthrie, David Hestenes, Priscilla Laws, Lilian Macmillan, R. Thorsten, Jack Wilson, and Dean Zellman. I would like to thank John Clement, Priscilla Laws, and Len Jessen for commenting on the manuscript. The suggestions of two very conscientious AIP reviewers were helpful in guiding me to make revisions that improved the paper significantly. Finally, I would like to thank my wife, Dr. Janice C. Redish, for numerous discussions on the subject of cognitive studies. Her expertise has played a major role in the development of my views on these issues. I would also like to thank her for her detailed editorial comments on the presentation. She is not to be held responsible for my persistent stylistic idiosyncrasies.

11. If we use the term "cognitive studies" as opposed to the term "cognitive science," which is more prevalent in the literature, advisable, the field is extremely broad and cuts across many disciplines. Very little in cognitive science satisfies the scientific criteria we are used to in physics of being precisely stated and well-established facts. It is only in the effort to raise the expectation of my scientific readers as to the nature of the information presented here, I use the weaker term (and use it in a slightly "pc" way."The other ones are not only of necessity..."
12. [There are a few notable exceptions. Piaget has made studies on how children build concepts of the physical world, and a number of trained physicists such as Fred Reid, John Clement, and John Moses, among others, could be counted as cognitive scholars.


D. J. Hoyt, "Learning the bucket principle," *J. Res. Sci. Teach.*, 13, 32-42 (Jan-Feb 1976), and references therein.


M. J. Johnson, "Metaphors We Live By" (University of Chicago Press, Chicago, 1979), 70.

D. J. Hoyt, "It is not a low fuel permits to the object we can still beat it up by doing work on it. The process is a component of the angular momentum. Most physicists can correctly interpret this abridgement without any difficulty.

In addition to giving them center on to build future learning, knowing a few things well gives the student a model of what it means to understand something in physics. This valuable point that has been frequently stressed by Arnold Newman. It is an essential element in the mental ecology of a scientist.

"Of course the student's mental model in this case is in fact correct. Lighter objects do fall more slowly than heavy ones if they fall at an air, and we also have much direct experience with objects falling in a vacuum, but that observation does not yield a useful idealization. The observation of objects of different masses fall in very nearly the same way.

Unfortunately, these mechanics appear to atrophy if unused.

