

Thursday, September 13th

- Sommerfeld's Refinements to the Bohr Model and Fine Structure
 - Multielectron Atoms
- Permitted and Forbidden Line Emission

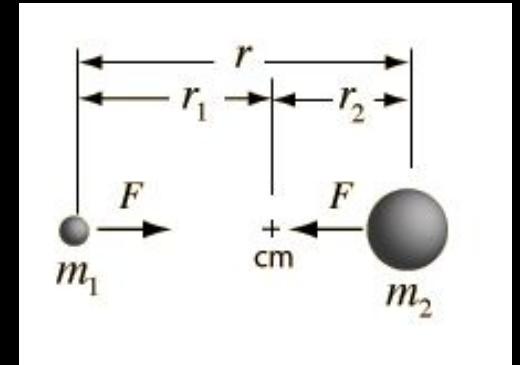
Data Reduction Review & Homework
Questions

Reduced Mass Correction

Reduced Mass:

$$m_e = \frac{m_p}{1836.153}$$

$$\mu = \frac{(m_e \cdot m_p)}{(m_e + m_p)} = \frac{1}{1823.8}$$



$$\mu = 5.4823 \times 10^{-4} \text{ amu} = 9.104589596 \times 10^{-28} \text{ g}$$

$$m_e - \mu = 4.793004302 \times 10^{-31} \text{ g}$$

Consequence for the Bohr Atom:

$$E_n = -\frac{2\pi^2 Z^2 e^4 m}{nh^2} = -\frac{2\pi^2 \cdot 1^2 \cdot (4.803204273 \text{ statcoulombs})^4 \cdot (4.793004302 \times 10^{-34} \text{ g})}{1 \cdot 6.6260755 \times 10^{-27} \text{ erg s}}$$

$$\delta E = .00716 \text{ eV}$$

$$\Delta \lambda = .4797 \text{ \AA}$$

$$\frac{\lambda}{\Delta \lambda} = 1.368 \times 10^4$$

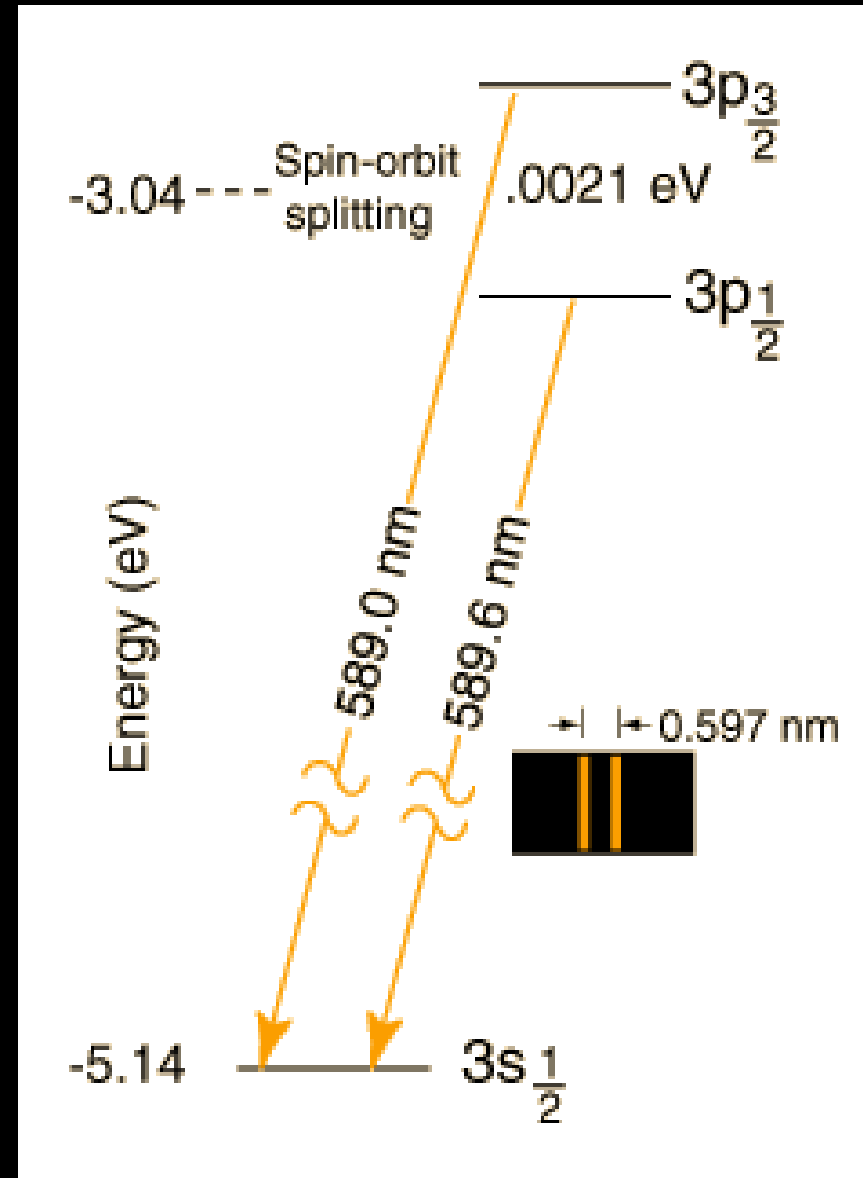
Is fine structure important?

Fine structure is very small in general, but significant for larger atoms with larger angular momenta

$$m_z B = m_B S_z L_z$$

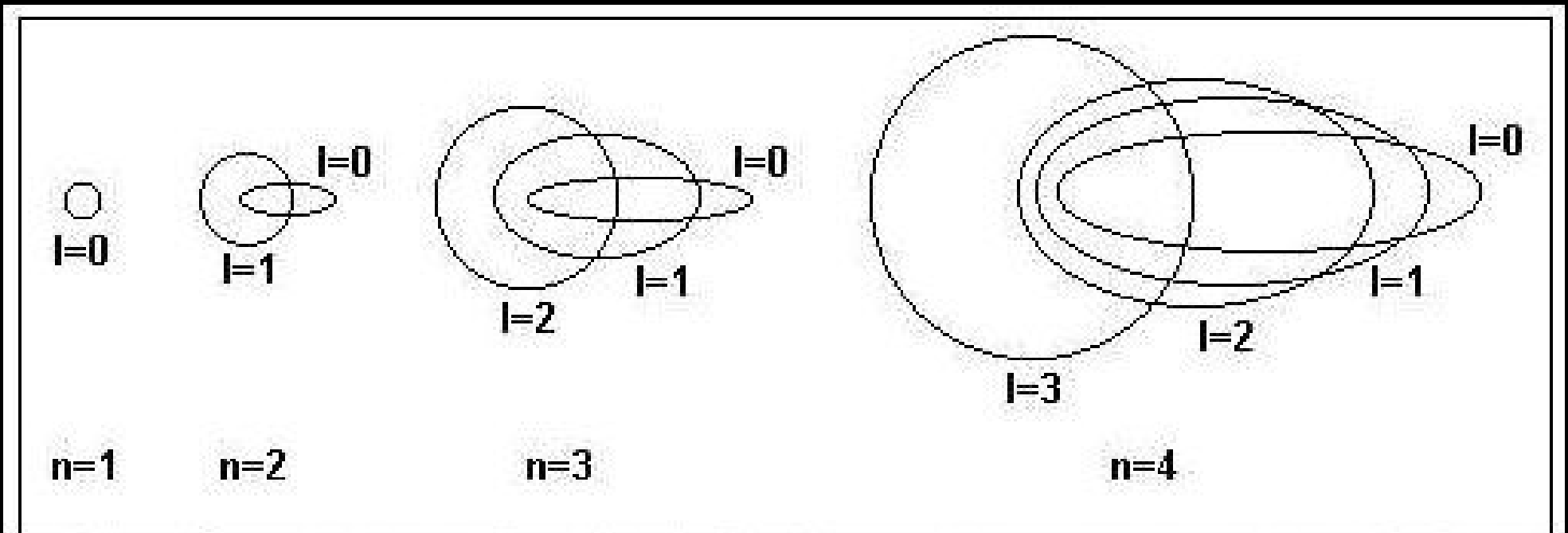
$$\Delta E = m_B g B$$

$$m_B = \frac{eh}{2m_e c \pi} = 5.7883817555 \times 10^{-5} \frac{\text{eV}}{\text{T}}$$



Sommerfeld's Correction

- Semiclassical attempt to correct for small errors in the Bohr model
- treats electron orbits as ellipses



The allowed electronic orbits for the four main quantum numbers by the Bohr-Sommerfeld model.

Sommerfeld Quantization

Instead of taking $mvr = nh$, which assumes a circular orbit, use the path integral around the ellipse (which is still angular momentum):

$$\int p ds = nh$$
$$E = U + K = \frac{Ze^2}{r} + \frac{p^2}{2m} = \frac{m}{2}(\dot{r}^2 + r^2 \dot{\phi}^2)$$
$$p_r = m\dot{r}, L_\phi = mr^2 \dot{\phi}$$

Since we're looking at an elliptical orbit, Kepler's 2nd law gives that angular momentum is conserved, so L is a constant over the integral, and therefore

$$\int L_\phi d\phi = 2\pi r^2 \dot{\phi} = 2\pi mvr = kh$$

i.e. angular momentum is quantized

Kepler



Sommerfeld Correction to Energy

There is no direct energy dependence on k , but “relativistic effects” cause the orbits to precess

$$E_n = \frac{-Z^2 e^4 \mu}{8 \epsilon_0^2 n^2 h^2} \left[1 + \frac{\alpha^2 Z^2}{n} \left(\frac{1}{k} - \frac{3}{4n} \right) \right]$$

Just as with the center-of-mass correction, we have a very small correction of order 4.5×10^{-5} eV or .1 angstroms

Multielectron Atoms

Energy calculated using approximations of Schrodinger equation called Hartree-Fock equations

- central potential of averaged nucleus and surrounding electrons

- "exchange potential" effective repulsion from spin because of Pauli Exclusion principle

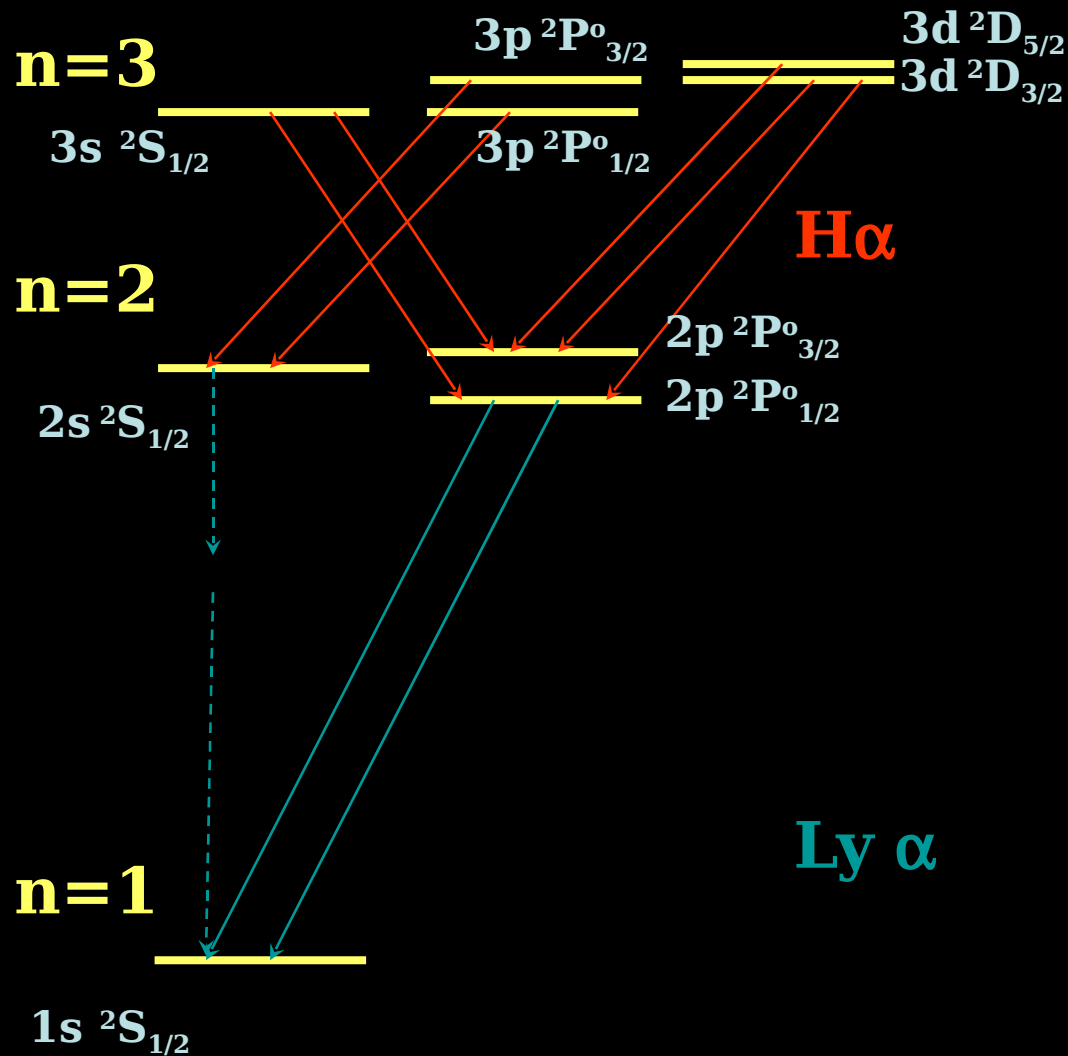
Hund's Rules (strictly only for ground state)

- larger spin => lower energy

- in a given configuration and spin, particles with the largest angular momentum have lowest energy

- Both result from electron repulsion: spin from Pauli Exclusion, angular momentum from electric repulsion

Permitted Transitions



Fine structure const.:

$$\alpha = e^2 / \hbar c = 1/137$$

Fine structure:

$$\delta E / E \sim \alpha^4 \sim 5 \times 10^{-5} \text{ eV}$$

Spin / orbit ($l * s$)

Hyperfine structure:

$$\delta E \sim 6 \times 10^{-6} \text{ eV}$$

Selection Rules:

$$\Delta l = 0, +/-1$$

$$\Delta j = 0, +/-1$$

even \Leftrightarrow odd parity

Forbidden Transitions

Notation:

O I 6300 - Permitted

Mg I] 4571 – Intercombination or
Semiforbidden

[O II] 7320 – Forbidden

How to use

<http://www.nofs.navy.mil/data/FchPix/cfra.html>

Desired Position Information

Field Name: NoName	E-mail for image notification (RECOMMENDED):		
Reset Form	Center:	RA (h m s) 00 00 00.0	Dec (d m s) 00 00 00.0
			Equinox J2000.0 Epoch 2000.0
Retrieve Data	Rectangle of Size:	10.0	10.0
	Circle of Radius:	10.0	.
			Minutes

Sources To Search

Catalogue(s): NOMAD USNO B1.0 USNO A2.0 ACT	Image Data: None All Surveys POSS-I (103aO, 103aE) POSS-II (IIIaJ, IIIaF, IV-N)
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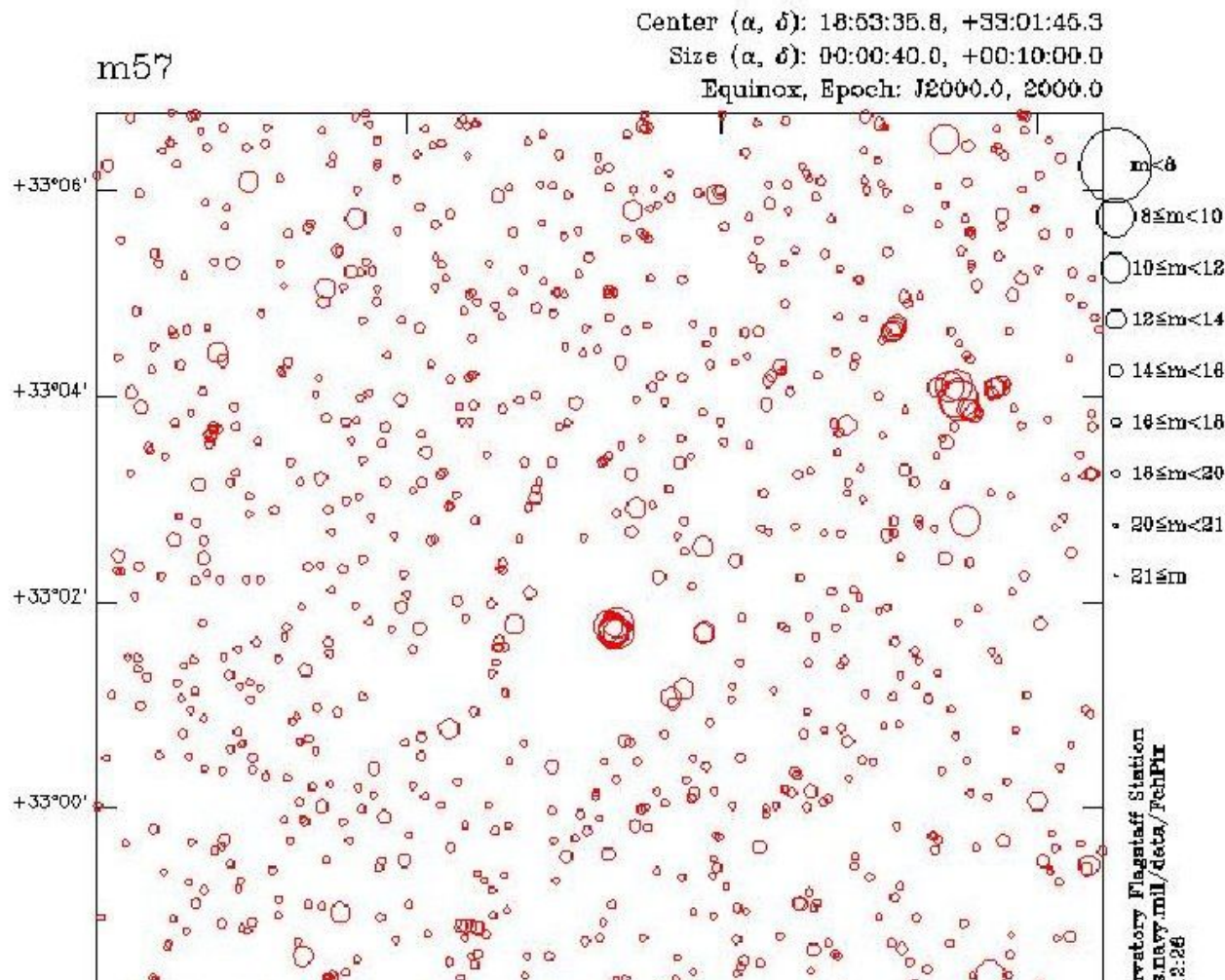
Types of Information to Be Returned

<input checked="" type="checkbox"/> Catalogue Lists						<input checked="" type="checkbox"/> Finder Charts					
						<input type="checkbox"/> Images					
<input type="checkbox"/> Get ALL Catalogue Columns (overrides check boxes below)											
Show	Sort	Column	Variable	Units/Format/Etc	Min	Max					
<input checked="" type="checkbox"/>		ID	id								
<input checked="" type="checkbox"/>		Alternate ID	altid								
<input checked="" type="checkbox"/>	<input type="checkbox"/>	RA	ra								
<input checked="" type="checkbox"/>	<input type="checkbox"/>	DEC	dec	hh/dd mm ss							
<input checked="" type="checkbox"/>	<input type="checkbox"/>	Sigma RA	sigra	[mas]							
<input checked="" type="checkbox"/>	<input type="checkbox"/>	Sigma DEC	sigdec	[mas]							
<input type="checkbox"/>	<input type="checkbox"/>	Mean Epoch	mep	[years]							
<input checked="" type="checkbox"/>	<input type="checkbox"/>	Mu RA	mura	[mas/yr]							
<input checked="" type="checkbox"/>	<input type="checkbox"/>	Mu DEC	mudec	[mas/yr]							
<input type="checkbox"/>	<input type="checkbox"/>	Mu Prob	muprob	[0.0 - 0.9]							
							Overplot Star Lists:	No			
							Format:	JPEG			
							Also Return FITS:	No			
							Orientation:	NW - North Up, East Left			
							Coordinate Overlay:	Tick Marks			
							Label Stars:	No			
							Center Mark:	None			

Immediate Results...

USNO Archive Search Results for Field m57

[Catalogue Info](#) | [Image Info](#)



The results you want to look at

Catalogue and Finder Chart Results

[Star Chart](#)

[USNO B1.0 Star List \(File Format Info\)](#)

Field Name: m57
Equinox, Epoch : J2000.0 2000.00
Field Center (RA,DEC): 18.8932750 , 33.0292500 [hours,degrees]
Field Size (RA,DEC): 0.1666667 x 0.1666667 [degrees]
Primary Magnitude: R2
Bright and Faint Mag. Limits: 0.00, 100.00
Catalogue Color: B2 - R2
Blue and Red Color Limits: -100.00, 100.00
Proper Motion Limits: 0.00, 10000.00 [milli-arcsec/yr]
Catalogue(s): USNO-B1
Star Lists are sorted in: RA
Epoch Date format: Julian Epoch (YYYY.YYYY)
Retrieve Images: Yes

Image Database Information

Schmidt Plates

	No.	Survey	Field	Plate	Date	Time	RA (B1950)	DEC RA (J2000)	DEC	Type	Filt	Exp	H.A.	dbID	Nimg
FITS jpg	001	POSSI	0336	267	1951.5086	06.20	4.894340	0.525247	4.900392	0.525890	103aE	RP2444	50 1:24E	se0336	0001
FITS jpg	002	POSSI	0336	267	1951.5086	07.15	4.894340	0.525247	4.900392	0.525890	103a0	None	12 0:27E	so0336	0001
FITS jpg	003	POSSII	0396	SF04745	1992.6366	04.10	4.921826	0.610863	4.929681	0.611892	IIIaF	RG610-3	70 0:32E	sf0396	0002
FITS jpg	004	POSSII	0396	SJ04083	1991.5332	06.93	4.921854	0.610862	4.929709	0.611891	IIIaJ	GG385-2	50 0:11E	sj0396	0002
FITS jpg	005	POSSII	0396	SN05857	1994.4627	07.60	4.921758	0.610850	4.929613	0.611878	IVN	RG9-8	60 1:13E	sn0396	0004
FITS jpg	006	POSSII	0458	SF02415	1989.3374	09.33	4.917523	0.523584	4.925960	0.524593	IIIaF	RG610-3	100 2:27E	sf0458	0001
FITS jpg	007	POSSII	0458	SJ02497	1989.5072	06.27	4.917495	0.523580	4.925931	0.524590	IIIaJ	GG385-2	45 1:27E	sj0458	0002
FITS jpg	008	POSSII	0458	SN06300	1995.5524	04.90	4.917421	0.523578	4.925857	0.524587	IVN	RG9-8	85 1:45E	sn0458	0004

N.B.: Remember that it will take at least 5 minutes (and probably closer to 10 or 15 minutes) for the requested pixels to be retrieved and placed on this web server. The more pixels that are requested, the longer this will take.

IRAF Review/Help

I can help more with IRAF and computers than QM and physics – I'll leave that to Bally.

Onto demo of:
imexamine

Homework 1 - Analysis

- Orientation of the image (to 1 degree precision).
- The pixel scale on the sky (in arcseconds per pixel).
- Knowing that the pixel size is 6.8 micrometers, calculate the effective focal length of the imaging system.
- Estimate the magnitude of the faintest star that is reliably detected on the image.
- Estimate the magnitude that corresponds to 1 count (ADU) on the image (the photometric 0-point).

I'm more concerned with how you choose to do these things than your actual answers – please include an explanation, if it's a reasonable attempt but a wrong answer you'll get more credit than a correct answer with no justification.