Astronomy 3520: Exposure Time Calculation

1. Spectral Resolution

$R = \frac{\lambda}{\Delta\lambda} = \frac{\nu}{\Delta\nu} = n\,N$ (Kitchin 8.15)

Note that this gives you $\Delta\nu = \frac{\nu}{\lambda}\Delta\lambda = \frac{c}{\lambda^2}\Delta\lambda$,

which also comes from the derivative of the equation $\nu = \frac{c}{\lambda}$

(except for the sign - an increase in $\nu \to$ a decrease in λ).

2. Photon Counts

$\frac{\text{Counts}}{\text{Second}} = \frac{\text{Flux} * \text{Area} * \Delta\nu}{h\,\nu} * \eta$

There are two ways to go about solving this using 'known' quantities:

I. Photon Energy

The energy of a photon is given by E = hν (see Kitchin eqn 1.4). If you plug in your central wavelength for the grating, you get the average photon energy*. You also should know the resolution R, from which you could calculate $\Delta\nu$ You should have looked up the flux of your source, possibly from its magnitude, and you know the area of the telescope is πr^2. The only remaining term is η. η_{Echelle} = .09, η_{DIS} = Improved?

- **Example:**

Using DIS's B1200, assuming it is 1mm long and therefore has a spectral resolution of N=1200, and ν_{central} = 4400 Å. Assume η = .2, using a magnitude 12 flux for V-band (which isn't really right, but I don't have any fluxes available for 4400Å)

$$R_{B1200} = 1200$$
$$\nu_{B1200} = \frac{\text{SpeedOfLight}}{4400\ \text{Angstrom}}$$
$$\Delta\nu_{1200} = \frac{\nu_{B1200}}{1200}$$
$$E_{B1200} = \text{PlanckConstant} \times \nu_{B1200}$$
$$\eta_{B1200} = .2$$
$$\text{CpS}_{B1200} = \frac{\text{Flux}_{M12} \times \pi\,(1.75\ \text{Meter})^2 \times \Delta\nu_{1200}}{E_{B1200}}\,\eta_{B1200}$$

$$\frac{1598.36}{\text{Second}}$$

II. Resolution + Planck's constant

Recalling our equation for spectral resolution, note that we have a term $\frac{\Delta\nu}{\nu}$ in our count equation. Plug in $\frac{1}{R}$ for that and you again have all known quantities in the equation

- **(same) Example:**

$$\text{CpS}_2 = \frac{\text{Flux}_{\text{M12}} \times \pi\, (1.75\,\text{Meter})^2}{\text{PlanckConstant} \times \text{R}_{\text{B1200}}}\, \eta_{\text{B1200}}$$

$$\frac{1598.36}{\text{Second}}$$

Hopefully you're not surprised that the answers are the same.

Detailed Example:

This is an in-depth example spreading the calculation into many separate parts to illustrate in more detail where each term comes from. In this calculation, I use some numbers that may not be familiar or necessarily correct, but they are approximately right (central wavelength for V band and zero-magnitude, i.e. Vega, flux in V band)

$$\text{R}_{\text{DIS}} = 3000$$
$$\text{R}_{\text{Echelle}} = 30\,000$$

$$\text{Jansky} = 10^{-26}\, \frac{\text{Watt}}{\text{Meter}^2\, \text{Hertz}}$$

$$\text{F}_{\text{V,Vega}} = 3953\, \text{Jansky}$$

$$\lambda_{\text{V}} = 5000\, \text{Angstrom}$$

$$\nu_{\text{V}} = \text{Convert}\left[\frac{\text{SpeedOfLight}}{\lambda_{\text{V}}}, \text{Hertz}\right]$$

$$\text{IncidentFlux}_{\text{DIS}} = \text{F}_{\text{V,Vega}} * \pi \left(\frac{3.5\,\text{Meter}}{2}\right)^2 * \frac{\nu_{\text{V}}}{\text{R}_{\text{DIS}}}$$

$$\frac{0.00076012\,\text{Erg}}{\text{Second}}$$

$$\text{IncidentFlux}_{\text{Echelle}} = \text{F}_{\text{V,Vega}} * \pi \left(\frac{3.5\,\text{Meter}}{2}\right)^2 * \frac{\nu_{\text{V}}}{\text{R}_{\text{Echelle}}}$$

$$\frac{0.000076012\,\text{Erg}}{\text{Second}}$$

Then, include instrument efficiencies:
Detector quantum efficiency ~.8 (conservatively)

$$\eta_{\text{QE}} = .8$$

Grating efficiency ~.1 for DIS, $.1^2$ for Echelle

$$\eta_{\text{DIS}} = .1$$

$$\eta_{\text{Echelle}} = .1^2$$

$DetectorFlux_{DIS} = IncidentFlux_{DIS} * \eta_{QE} * \eta_{DIS}$

$\frac{0.0000608096\ Erg}{Second}$

$DetectorFlux_{Echelle} = IncidentFlux_{Echelle} * \eta_{QE} * \eta_{Echelle}$

$\frac{6.08096 \times 10^{-7}\ Erg}{Second}$

Now we know the incident flux per frequency element, so we must make some calculations related to the detector sensitivty. Specifically, we need to know how many counts per second correspond toa given energy per second.

$CountsPerEnergy = \frac{1}{PlanckConstant * \nu_V}$

$\frac{0.403277}{ElectronVolt}$

$CountsPerSecond_{DIS} = DetectorFlux_{DIS} * CountsPerEnergy$

$\frac{1.53061 \times 10^{7}}{Second}$

$CountsPerSecond_{Echelle} = DetectorFlux_{Echelle} * CountsPerEnergy$

$\frac{153061.}{Second}$

Then determine how many counts you need. You want a strong signal that is well below the detector nonlinearity region. For DIS, a good choice is probably 30,000 counts for a deep image. For Echelle, you probably can't expect as many, but you need to make sure you have many more than the read noise. However, for Vega, you're going to get a lot of counts - it's very, very bright for a 3.5m aperture.

$Convert[30000 / CountsPerSecond_{DIS}, Minute]$

0.0000326666 Minute

$Convert[5000 / CountsPerSecond_{Echelle}, Minute]$

0.000544444 Minute

A more reasonable calculation is for a 12th magnitude source:

$Flux_{M12} = F_{V,Vega} * 10^{-12/2.512}$

$\frac{6.60476 \times 10^{-28}\ Watt}{Hertz\ Meter^2}$

$CpS_{DIS,12} = Convert\left[\frac{Flux_{M12} * \pi\ (1.75\ Meter)^2}{R_{DIS} * PlanckConstant}\ \eta_{DIS},\ 1 / Second\right]$

$\frac{319.673}{Second}$

$$CpS_{Echelle,12} = \text{Convert}\left[\frac{Flux_{M12} * \pi\,(1.75\,\text{Meter})^2}{R_{Echelle} * \text{PlanckConstant}}\,\eta_{Echelle},\ 1/\text{Second}\right]$$

$$\frac{3.19673}{\text{Second}}$$

```
Convert[30 000 / CpS_{DIS,12}, Minute]
```

1.5641 Minute

```
Convert[5000 / CpS_{Echelle,12}, Minute]
```

26.0683 Minute

Explanation of *:

These calculations are all still incorrect. We have left out atmospheric attenuation in our η, which will decrease the count rate and therefore increase our exposure time. However, the atmospheric attenuation is variable (e.g. with airmass, turbulence, amount of water), and we are not concerned with extremely precise measurements of photon counts as a result. We are only trying to get a pretty good approximation using reasonable assumptions that will tell us about how long to expose. We usually end up taking exposures in integer minutes (e.g. 5 minutes, 10 minutes...) because that's the easiest way to divide up a night of observing. That approximation only really breaks down in the case that you risk overexposing - so if we take a look at Vega, for example, we'll probably use the shortest exposure time available.