ASTR 3520, Fall 2005
Homework Set #2:
Black-Bodies, Flux from an object, and the spectrum of hydrogen
Due Thursday, 29 September, 2005

Luminosity, brightness, and magnitude of a star or planet. We will assume that stars radiate like blackbodies.

Working with the Planck Function.

(1) What are the frequencies, ν, corresponding to wavelengths of 0.5 μm and 10 μm?

(2) Consider an object with a temperature of 10^4 K. At what wavelength does $B_\nu(T)$ peak?

(3) Consider an object with a temperature of 300 K. At what wavelength does $B_\nu(T)$ peak?

(4) Use the blackbody formula, $B_\nu(T)$ to calculate the emitted flux from each square centimeter of a black-body radiator at wavelengths of 5,000Å into one hemisphere assuming that the radiator has a temperature of 10,000 Kelvin.

(5) A monochromatic luminosity is the product of the radiating surface area times $B_\nu(T)$. What are the monochromatic luminosities in $erg\ s^{-1}Hz^{-1}$ emitted into a hemisphere in a passband of 1 Hz of a 10,000 K star which has a radius of 10^{11} cm at these wavelengths? (Note: This corresponds roughly to a star of spectral type A, similar to Vega).

(6) Show that if $\Delta\lambda$ is the passband of a filter in wavelength units, the corresponding passband in frequency units $\Delta\nu$ is given by $\Delta\nu/\nu = \Delta\lambda/\lambda$.

(7) What is the luminosity (erg sec^{-1}) of this (from problem 5) star in a passband of $\Delta\lambda = 1{,}000$ Å wide centered at 5,000 Å.

(8) Approximately how many photons are produced each second by this star in these passbands (the photon luminosity as opposed to ergs per second)?

(9) If the star is located at a distance of 100 pc, what are the fluxes of energy (erg s^{-1} cm^{-2}) and of photons (photons s^{-1} cm^{-2}) in this passband? Note that the flux here is at observer and not at the surface of the star.

(10) What is the apparent magnitude of the star in problem 9? Note: At 5,000Å, a star with an apparent magnitude of 0 (e.g. Vega). has a flux density of about 4,000 Jy. (Definition of a Jansky - 1 Jy = 10^{-26} Watts m^{-2} Hz^{-1}. This is a standard unit in radio astronomy, but is coming into common usage at all wavelengths. It is named after Karl Jansky, who in 1929 discovered the first cosmic radio waves with an antenna in Holmdel NJ).

(11) Suppose you observe this star with a 24 inch telescope with a throughput efficiency (including the CCD quantum efficiency) of 50%, and that all of the star's light falls onto 1 pixel of a CCD camera. What is the rate at which photo-electrons are generated in the CCD pixel?

The Hydrogen Atom

Ionization of H represents a transition from the $n = 1$ level of hydrogen to the $n = \infty$ level. Any photon with an energy of $E\ > 13.6$ eV can ionize hydrogen.

Transitions in hydrogen occur when the quantum number n changes. The emission spectrum is produced then an upper state, n_u, decays into a lower state, n_l. The the resulting energies of the emitted photons are given by

$$E(n_u, n_l) = R(1/n_l^2 - 1/n_u^2)$$

(12) Using the above facts, evaluate R in c.g.s units. Hint: set $n_u = \infty$, $n_l = 1$.

(13) The Balmer series of H emission lines occurs when the an upper state decays into $n_l = 2$ state. Thus, $n_u = 3 \Rightarrow n_l = 2$ corresponds to the Hα line. $n_u = 4 \Rightarrow n_l = 2$ corresponds to the Hβ line. $n_u = 5 \Rightarrow n_l = 2$ corresponds to the Hγ line.

Calculate the wavelengths in Angstroms of these lines.

(14) The Lyman series of H emission lines occurs when the an upper state decays into $n_l = 1$ state. Thus, $n_u = 2 \Rightarrow n_l = 1$ corresponds to the Lyα line. $n_u = 3 \Rightarrow n_l = 1$ corresponds to the Lyβ line. $n_u = 4 \Rightarrow n_l = 1$ corresponds to the Lyγ line. $n_u = \infty \Rightarrow n_l = 1$ corresponds to the Lyγ line.

Calculate the wavelengths in Angstroms of these lines.

(15) Plot the spectrum of the Lyman and Balmer lines of H. Show where the ionization of the Lyman series and Blamer series occurs (the Balmer edge corresponds to $n_u = \infty$, $n_l = 2$).