

Estimating Exposure Times

Brian Keeney

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This document demonstrates how you can determine the brightness of an object that is detected at a particular signal-to-noise ratio (S/N) in images and spectra of a given exposure time.

Flux of Vega

The first step is to calculate the flux of Vega at the wavelength of interest, which will define the flux of an object with $m = 0$. Like all A0 stars, Vega's spectrum can be approximated by that of a blackbody with a temperature of 10,000 K. The intensity of blackbody radiation is given by the Planck function:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \left(e^{h\nu/kT} - 1 \right)^{-1}. \quad (1)$$

The relationship between the emitted flux of a blackbody at any two frequencies is

$$\frac{F_{\nu_1}}{F_{\nu_2}} = \frac{B_{\nu_1}(T)}{B_{\nu_2}(T)} = \frac{\nu_1^3 (e^{h\nu_1/kT} - 1)^{-1}}{\nu_2^3 (e^{h\nu_2/kT} - 1)^{-1}}. \quad (2)$$

We know from class notes that Vega has a flux of 4000 Jy at $\lambda = 5000 \text{ \AA}$, so the flux of Vega at any frequency is given by

$$F_{\nu_{14}}(\text{Jy}) = \frac{312 \nu_{14}^3}{e^{0.480 \nu_{14}} - 1} \quad (3)$$

where ν_{14} is the frequency of interest in units of 10^{14} Hz and $F_{\nu_{14}}$ is in Janskys.

Imaging

Assume that we are imaging a star in V-band with the SBO 24-inch telescope. The V-band is centered at $\lambda \approx 5500 \text{ \AA}$ and has a width of $\Delta\lambda \approx 1000 \text{ \AA}$. Using Equation 3 above we find that Vega has a flux of 3980 Jy at $\lambda = 5500 \text{ \AA}$, and using the relation from HW2 that $\Delta\nu = \nu(\Delta\lambda/\lambda)$ we find the the total flux from Vega in V-band is $F = F_\nu \Delta\nu = 1.1 \times 10^6 \text{ photons s}^{-1} \text{ cm}^{-2}$. This is the flux of photons that reach Earth from Vega, but the atmosphere is not completely transparent so only about 80% of the V-band photons that reach Earth will make it through the atmosphere to our telescope. Thus, the number of photons that reach our telescope from Vega is $0.8(1.1 \times 10^6) = 8.8 \times 10^5 \text{ photons s}^{-1} \text{ cm}^{-2}$.

The 24-inch telescope has a collecting area of $A = \pi R^2 = \pi(12 \text{ in.} \times 2.54 \text{ cm/in.})^2 = 2920 \text{ cm}^2$, which implies that $FA = 2.6 \times 10^9$ photons from Vega strike the primary mirror each second. Not all of these photons reach the detector, however. About 10% of the light is lost with each reflection due

to vignetting, or the shadowing of the primary mirror by the secondary and its support structure. Approximately another 10% of the photons are lost at each reflection due to imperfect reflectivity of the mirrors from the combination of material limitations and dust. After reflecting off of the primary and secondary mirrors, the incident photons will reflect one last time off of the flip mirror that sends them to the imaging CCD rather than allowing them to enter the spectrograph. Once there, the V-band filter will only transmit about 90% of the incident photons. The transmitted photons will then strike the CCD, but the QE of the SBIG ST 1001-E CCD camera is ~ 0.6 at $\lambda = 5500 \text{ \AA}$, so only about 60% of these photons will be detected. All of these losses imply that the SBO 24-inch telescope and imaging CCD have an end-to-end throughput of $(0.9 \cdot 0.9)^3 \cdot 0.9 \cdot 0.6 \approx 0.3$, meaning that only 30% of the V-band photons that strike the telescope make it through the system to be detected! This is actually a fairly typical number and a similar calculation for the APO 3.5-m telescope yields the same result.

These telescope losses imply that the 24-inch CCD detects $0.3(2.6 \times 10^9) = 7.7 \times 10^8$ photons s^{-1} from Vega. Since the CCD gain is $3.1 \text{ e}^-/\text{ADU}$, if each detected photon creates one photoelectron then the measured flux of Vega is $2.5 \times 10^8 \text{ ADU s}^{-1}$. Seeing will distribute these counts over several pixels on the detector. The 24-inch imaging CCD has a pixel scale of about $1 \text{ arcsec pix}^{-1}$ and the typical seeing at SBO is $\sim 3''$. A Gaussian distribution will contain 75% of the total light of a star within 1 FWHM of the centroid. With $3''$ seeing and 1 arcsec^2 pixels, this means that 75% of Vega's light will be spread out over $\pi 1.5^2 = 7$ pixels, yielding an average count rate in this region of $2.7 \times 10^7 \text{ ADU s}^{-1} \text{ pix}^{-1}$. This means that Vega, or any zeroth magnitude star for that matter, will saturate the detector in 0.002 seconds (less if the seeing is better!).

The final consideration is to estimate the level of the sky background that you will be measuring your signal against. Boulder has particularly bright skies that have a V-band surface brightness of $\sim 19 \text{ mag arcsec}^{-2}$. Dark sites like Apache Point have V-band sky brightnesses closer to $21.8 \text{ mag arcsec}^{-2}$, more than 10 times fainter than Boulder skies. A V-band surface brightness of $19 \text{ mag arcsec}^{-2}$ implies that each square arcsecond of sky is $10^{0.4 \cdot 19}$ times fainter than the V-band flux of Vega and therefore has a count rate of $(2.5 \times 10^8) 10^{-0.4 \cdot 19} \approx 6 \text{ ADU s}^{-1} \text{ arcsec}^{-2}$. Since the 24-inch imaging CCD has 1 arcsec^2 pixels, this translates into a sky brightness of $6 \text{ ADU s}^{-1} \text{ pix}^{-1}$.

Now we have everything we need. Ignoring errors introduced by dark/bias subtraction and flat fielding, the S/N in the measurement of a star's brightness is

$$S/N = \frac{N_*}{\sqrt{N_* + 2N_{\text{sky}}}}, \quad (4)$$

where N_* is the total star counts and N_{sky} is the sky counts in the measurement aperture. We can rearrange Equation 4 to solve for N_*

$$N_* = 0.5 \left((S/N)^2 + \sqrt{(S/N)^4 + 8N_{\text{sky}}(S/N)^2} \right), \quad (5)$$

which yields the number of star counts required to measure an object's brightness to a given S/N.

As an example, let's calculate the number of counts necessary to detect a star at $S/N = 5$ (the faintest object whose brightness can be measured with any reliability) in a 5 minute image (the longest practical exposure time until we get the autoguider working) taken with the SBO 24-inch telescope. The sky counts will be $6 \cdot 300 = 1800$ ADU pix^{-1} , but N_{sky} is the total sky counts in the measurement aperture. Let's assume as before that we have $3''$ seeing and additionally that we use a measurement aperture with a diameter of 3 FWHM, which will gather $> 99.9\%$ of the stellar light given a Gaussian distribution. Thus, the measurement aperture covers $\pi 4.5^2 = 64$ arcsec $^2 = 64$ pixels and $N_{\text{sky}} = 1800 \cdot 64 = 1.1 \times 10^5$ ADU. Plugging this value and $S/N = 5$ into Equation 5 yields $N_* = 2400$ ADU. An object of this brightness is $300(2.5 \times 10^8)/2400 = 3.1 \times 10^7$ times fainter than Vega, which means that it has an apparent V-band magnitude of $m \approx 19$.

Now let's assume we want to observe a nebula or galaxy. In this case the seeing doesn't really matter so long as the object of interest has a relatively uniform surface brightness over scales of a few arcseconds (this is a better approximation for nebulae than galaxies, but good enough in both cases for this calculation). The only real difference between this case and the stellar case is that now we want a S/N **per pixel** of 5 and the N_{sky} we plug into Equation 5 and the N_* that it returns are also measured per pixel. So if we assume as above that the sky brightness in Boulder is 19 mag arcsec $^{-2}$ and an exposure time of 5 minutes, then we can plug $S/N = 5$ and $N_{\text{sky}} = 1800$ ADU pix^{-1} into Equation 5 to find $N_* \approx 310$ ADU pix^{-1} . Now we can convert this into a surface brightness by using the fact that we have 1 arcsec 2 pixels to find $N_* \approx 310$ ADU arcsec $^{-2}$. This value is $300(2.5 \times 10^8)/310 = 2.4 \times 10^8$ times fainter than Vega, meaning that we reach a limiting V-band surface brightness of 21 mag arcsec $^{-2}$ in a 5 minute exposure taken at SBO.

Spectra

The main difference between spectral and imaging exposure time estimates is that you need to carry the Hz^{-1} flux dependence through until the end for the spectral calculation. This time, let's assume that we're interested in estimating the $\text{H}\alpha$ flux of a star, galaxy, or nebula. Equation 3 tells us that the flux of Vega at 6563 Å is 3740 Jy $= 1.2 \times 10^{-8}$ photons $\text{s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$. We're still in the optical, so the atmosphere will absorb 20% of this light, leaving $0.8(1.2 \times 10^{-8}) = 9.9 \times 10^{-9}$ photons $\text{s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$ to reach the telescope.

Multiplying this value by the area of the SBO 24-inch telescope gives us the rate at which $\text{H}\alpha$ photons from Vega are incident on the primary mirror: $2920(9.9 \times 10^{-9}) = 2.9 \times 10^{-5}$ photons $\text{s}^{-1} \text{Hz}^{-1}$. We have all of the same telescope losses as before due to imperfect reflectivity of the mirrors and vignetting, but the spectrograph has a lot more optical components (collimator lens, grating, camera lens) that the incoming light must interact with before it strikes the CCD, bringing the end-to-end throughput of the telescope and spectrograph down to more like 10% than the 30% we found for the telescope and imaging CCD. Thus, only $0.1(2.9 \times 10^{-5}) = 2.9 \times 10^{-6}$ photons $\text{s}^{-1} \text{Hz}^{-1}$ are detected by the spectrograph CCD.

Grating 1 has a dispersion of ~ 0.5 Å pix^{-1} . Again using the relation from HW2 that $\Delta\nu =$

$\nu(\Delta\lambda/\lambda)$, this means that each pixel is 3.5×10^{10} Hz wide in the dispersion direction and the detected flux of Vega is 10^5 photons s^{-1} at 6563 Å. The ST-8 spectrograph CCD has a gain of $2.5 e^-/\text{ADU}$, so this corresponds to a count rate of 4×10^4 ADU s^{-1} . If we assume the typical SBO seeing of $3''$, then 75% percent of this light will be spread over 8 pixels in the spatial direction of the CCD (the ST-8 CCD has a pixel scale in the spatial direction of 0.38 arcsec pix^{-1}), meaning that the average count rate in these bright pixels goes down to 3750 ADU s^{-1} pix^{-1} .

We can still use Equation 5 to estimate S/N, but for spectra the sky level is concentrated in discrete spectral lines and so long as you are not trying to observe a feature at one of those wavelengths, the background in spectra is not nearly as bright as in images. Dark sites measure sky brightnesses in the optical between sky lines of 1.5×10^{-17} photons s^{-1} cm^{-2} Hz^{-1} arcsec^{-2} . Let's assume that skies at SBO are about 10 times brighter than this. After multiplying by the telescope collecting area, end-to-end throughput, and the number of Hertz per pixel in the dispersion direction we find that the SBO sky brightness at 6563 Å is 1.5×10^{-3} photons s^{-1} arcsec^{-2} = 6.1×10^{-4} ADU s^{-1} arcsec^{-2} . Finally, each pixel only covers $0''.38$, so the measured sky brightness is 2.3×10^{-4} ADU s^{-1} pix^{-1} . So in a 5 minute spectrum, we only collect 0.07 sky counts per pixel in between spectral lines!

Say we want to reach at least a S/N of 5 per pixel at the wavelength of $\text{H}\alpha$ in a 5 minute spectrum of a star. Assuming $3''$ seeing, we will use an extraction aperture that is $9'' \approx 24$ pixels wide, which will have a total of $N_{\text{sky}} = 24 \cdot 0.07 = 1.7$ counts from sky background. Equation 5 then tells us that we need a total of $N_* = 28$ counts in the stellar continuum at 6563 Å. This star is $300 \cdot 4 \times 10^4 / 28 = 4.3 \times 10^5$ times fainter than Vega, which corresponds to an apparent magnitude of $m \approx 14$.

If you're looking at a galaxy or nebula instead, then again the seeing doesn't matter and you're interested in the S/N and sky counts per pixel. In a 5 minute exposure, $N_{\text{sky}} = 0.07$ ADU pix^{-1} and Equation 5 gives us $N_* = 25$ ADU pix^{-1} at S/N = 5 (this makes sense since $N_{\text{sky}} \approx 0$). This corresponds to 66 ADU arcsec^{-2} , which is $300 \cdot 4 \times 10^4 / 66 = 1.8 \times 10^5$ times fainter than Vega at 6563 Å. Thus the limiting surface brightness in 5 minute spectra taken at SBO is about 13 mag arcsec^{-2} .

Limiting Magnitudes

The following table lists approximate limiting V-band magnitudes and surface brightnesses for images and spectra taken with the SBO 24-inch telescope, as well as the Apache Point 3.5-m telescope.

	SBO		APO	
	Images	Spectra	Images	Spectra
Sky Brightness (10^{-17} photons s^{-1} cm^{-2} Hz^{-1} arcsec^{-2})	25	15	1.9	1.5
Seeing (arcsec)	3	3	1	1
Exposure Time (min)	5	5	20	20
Limiting S/N	5	5	5	5
Limiting V-band Magnitude (mag)	19	14	25	21
Limiting V-band Surface Brightness (mag arcsec^{-2})	21	13	24	21