CHAPTER 6: SYNCHROTRON RADIATION

6.1 THE SYNCHROTRON FREQUENCY

Synchrotron radiation is, very simply, radiation from relativistic electrons moving in a uniform magnetic field. It is the relativistic equivalent of cyclotron radiation and is named after the relativistic accelerators used by physicists. When cyclotrons became sufficiently powerful to boost an electron close to the speed of light, the mass of the electron changed and so did its orbital frequency. As a result, the synchrotron had to adjust its boost frequency as the energy of the beam particles rose. These synchrotrons are in regular use around the world as a copious supply of ultraviolet and x-ray photons.

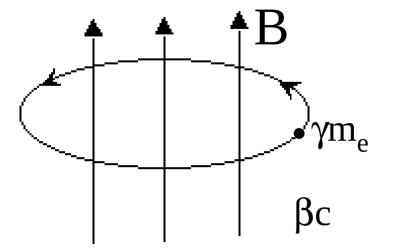


Figure 6.1: As a particle moves in a magnetic field, it has a circular orbit, even if relativistic.

The universe creates relativistic electrons and traps them in magnetic fields in a variety of different environments. The objects that emit the high energy electrons tend to be rather exotic and interesting, so understanding the nature of synchrotron radiation is of basic value to modern astrophysics.

Consider a relativistic particle of mass γm_e moving at velocity v perpendicular to the magnetic field lines. We can then write the equation for balance of centrifugal force against the Lorentz force in the relativisitic case. The right side of the equation was already in relativistic form in equation 5.1. Now we have:

$$\frac{d\vec{p}}{dt} = e\frac{\vec{v}}{c}\times\vec{B} \qquad (6.1)$$

which becomes

$$\frac{d}{dt}(gm_e v) = e\frac{v}{c}B \qquad (6.2)$$

proceeding in the usual way for an orbit we find that

$$gm_e\frac{v^2}{r} = e\frac{v}{c}B \qquad (6.3)$$

then solving for the orbital frequency we find

$$w = \frac{eB}{gm_e c} = \frac{w_c}{g} \qquad (6.4)$$

Thus the period of the orbit of the electron will drop by a factor of γ.

At first one might be tempted to assume that the radiation will fall in frequency accordingly. However, exactly the opposite is true. Consider the electron from an almost co-moving (i.e. non-relativistic) frame. The exact frame is irrelevant, as the cyclotron frequency is independent of the particle's velocity in the non-relativistic frame. However, due to time dilation effects as we shift frames, the particle will cross a factor of γ more magnetic field lines per second than in the lab frame, effectively increasing the field by a factor of γ. Thus in the non-relativistic frame we find the frequency of emission to be:

$$n = g \frac{eB}{mc} \tag{6.5}$$

Returning to lab frame, the radiation frequency is Doppler shifted by a factor of γ. If the particle is moving away from us, then the frequency shifts back down to as low as the original cyclotron frequency. If the particle is heading toward us, then the frequency shifts upward another factor of γ, leading us to:

$$n_s = \frac{g^2 eB}{2pm_e c} = \frac{4.8x10^{-10} g^2 B}{2px0.91x10^{-27}\,x3x10^{10}} = 2.8\gamma^2 Bx10^6 Hz = 2.8\gamma^2 B\ MHz \tag{6.6}$$

We refer to this frequency as the "synchrotron frequency" as it is the characteristic frequency of emission of the relativistic electron. In actual fact, the emission is now spread across a broad band of the spectrum, from essentially zero frequency up to the syhnchrotron. But, since it is the high energy photons that carry the bulk of the energy, we use

$$n_{sync} = 4x10^6 Bg^2 Hz \tag{6.7}$$

as the synchrotron characteristic frequency.

The actual distribution of the synchrotron radiation from an electron is shown in Figure 6.2. From detailed derivation of the physics, we know it to be a modified Bessel function when averaged over a full orbit. However, in astrophysical environments, we always have a range of energies (γ's) and this function is washed out. We simply concentrate on the characteristic energy of the emission.

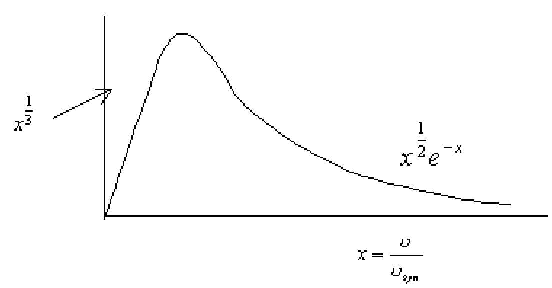
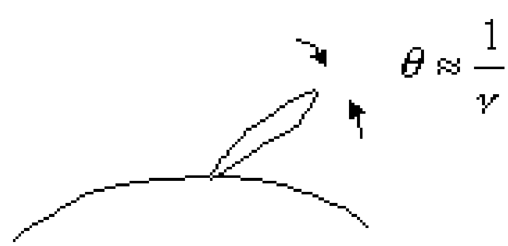


Figure 6.2: The actual spectral distribution of synchrotron emission from a single particle is a modified Bessel function. Because of group effects, we rarely worry about this function in astrophysics.

An electron experiences an upshift in radiation due to the relativistic transformation, but also experiences the famous "headlight effect". The radiation is primary emitted in the forward direction as



EXAMPLE

Electrons with energy of 50MeV are trapped in a region of space with a 10^{-4}Gauss magnetic field. What is γ? What will be characteristic frequency at which the electrons will emit?

Answer: γ=50MeV/511keV=100

$$n = 4x10^6 Bg^2 = 4x10^6 x10^{-4} x100^2 = 4MHz$$

the electron moves around its orbit. The angular width of the radiation is about 1/γ radians. Thus we expect, even in an astrophysical setting, for the radiation to be strongest in the plane perpendicular to the magnetic field.

6.2 ENSEMBLES OF ELECTRONS

The theory for the emission from a single electron in a uniform magnetic field is elegant and complete. However, one electron is too faint to observe at a large distance, so we always observe regions where large numbers of electrons are emitting together. These ensembles tend to have certain characteristics that allow us to analyze the physical parameters of the emitting regions.

Synchrotron emitters have highly non-thermal energy distributions. This is because the process that creates them is usually related to electro-magnetic phenomena in some way. We usually find that the distribution of electron energy can be given by a power law

$$N(E)dE = KE^{-a}dE \tag{6.8}$$

where N(E) is the is the number of electrons between E and E+dE.

When this power law of energies is convolved with the emission spectrum of an individual electron, we find that the output is also a power law.

$$j_n = 0.31*(0.24)^{-\frac{(a-1)}{2}} \frac{e^3}{mc^2}\left(\frac{3e}{4pm^3c^5}\right)^{\frac{a-1}{2}} B^{\frac{a+1}{2}} Kn^{-(\frac{a-1}{2})} \tag{6.9}$$

or more simply, jν, the emissivity per unit volume is given by

$$j_n \approx n^{-\frac{(a-1)}{2}} \tag{6.10}$$

Thus we usually see power law spectra emitted by synchrotron sources. From equation 6.9 we see that the intensity related to the number of electrons (from K) and the strength of the magnetic field, however, this information may be hidden in the intensity of the signal. The power law index α, which is defined by the distribution of electron energies is directly reflected in the power law index of the emitted spectrum.

6.3 SYNCHROTRON DECAY TIME

Just as in cyclotron radiation, we can estimate the decay time of a synchrotron source, and we start in the same way, with the Larmor formula, only this time we must adjust the Larmor formula with the addition of the γ^4.

$$P = \frac{2e^2 a^2 g^4}{3c^3} \tag{6.11}$$

Using the force balance equation in relativistic form we have

$$gma = e\frac{v}{c}B \tag{6.12}$$

which simplifies to

$$a = \frac{eB}{gm} \tag{6.13}$$

after substitution we find

$$P = \frac{2e^4}{3m^2c^3}B^2 g^2 \tag{6.14}$$

inserting the constants of nature

$$P = \frac{2*(4.8*10^{-10})^4}{3*(0.91*10^{-27})^2*(3*10^{10})^3}B^2 g^2 \text{ ergs/s} \tag{6.15}$$

after the algebra we find that a single electron emits power P:

$$P = 1.6 * 10^{-15} B^2\gamma^2 \text{ ergs/s} \tag{6.16}$$

The same electron has energy E:

$$E = \gamma mc^2 = 0.91*10^{-27} * 9*10^{20}\ \gamma \text{ ergs} \tag{6.17}$$

or

$$E = 8.2*10^{-7}\ \gamma \text{ ergs} \tag{6.18}$$

To find the characteristic time for loss of energy we divide the energy

$$t = \frac{E}{P} = \frac{8*10^{-7} g}{1.6*10^{-15}B^2 g^2} \tag{6.19}$$

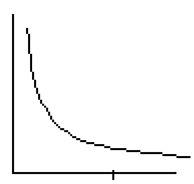
Thus the synchrotron decay time is given by:

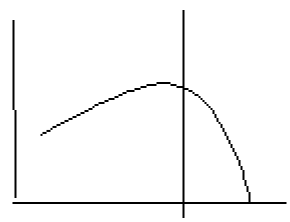
$$t = \frac{5x10^8}{B^2 g} s = \frac{17}{B^2 g} yrs \qquad (6.20)$$

Thus the decay time scales as $B^2\gamma$, while the frequency scales as $B\gamma^2$. This difference allows one to solve for both the magnetic field and the particle energy. If the decay time is not measurable, then an estimate of the magnetic field needs to be made. One way to do this is to compare the total magnetic field energy to the total particle energy and choose the field that leads to the smallest total energy. This approach is used to estimate the energy in the lobes of radio galaxies.

EXAMPLE

A radio astronomer is observing a source that is believed to be emitting by synchrotron radiation. The source fades away over a one month timescale. Its spectrum has a characteristic value of 1GHz, as shown below. Derive the magnetic field and electron energy (γ) of the emitting particles.





Answer: Use equation 6.20. The timescale is about $2x10^6$ seconds:

$$3*10^6 = \frac{5*10^8}{B^2 g} \text{ so that } B^2 g = 170$$

Then use the equation 6.7 for the synchrotron frequency

$$10^9 = 4*10^6 B g^2 \text{ so that } b g^2 = 250$$

Solving simultaneously we find that B ~ 5 gauss and γ ~ 7

EXERCISES:

1. The Crab Nebula is observed to emit x-rays having an energy of at least 100keV from an extended region. Compute the energy, lifetime, and Larmor radius of the electrons producing this radiation. Assume it is due to synchrotron radiation in a magnetic field of 10^{-4} gauss.

2. The nonthermal spectrum of the Crab exhibits a downward turn above 10^{15} Hz. Assuming this bend is due to lifetime losses, use the known age of the Crab (supernova in 1054 AD) to estimate the magnetic field strength.

3. A radio galaxy has created a giant lobe, 100,000pc in diameter, filled with relativistic electrons. The lobe emits $3x10^{40}$ ergs/sec of synchrotron emission at about 1GHz.

a) What is the energy of a typical electron (in ergs)?

b) How much energy is stored in the relativistic electrons of the lobe?

c) How long can the lobe survive without new electrons?

d) How much energy is stored in magnetic field? (Use $B^2/8\pi$)