

# Off-plane grating mount tolerances for Constellation-X

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## Abstract

High groove density reflection gratings placed at grazing incidence in the extreme off-plane mount offer increased performance over conventional in-plane mounts in the x-ray. We are developing an off-plane approach to the Reflection Grating Spectrometer of the Constellation-X Mission. In this paper we discuss the geometry of the off-plane mount and present formulae for the key tolerances of the grating array.

**Keywords:** Off-plane mount, holographic grating, x-ray spectroscopy

## 1. Introduction

### 1.1 Constellation-X

The Constellation-X mission will provide an instrument with large effective collecting area and high spectral resolution relative to past and current x-ray observatories. An overview of the mission and science goals is given by Tananbaum, et al.<sup>1</sup>, and can be found on the web at <http://constellation.gsfc.nasa.gov>.

Briefly, the mission incorporates three instruments, all designed to perform high throughput spectroscopy of astronomical x-ray sources. For x-ray energies above 10keV there is a hard x-ray telescope (HXT) and from 0.1 to 10keV, there is a large foil optic telescope known as the SXT. The SXT has its light divided between two instruments – a calorimeter and a reflection grating spectrometer (RGS). The calorimeter is optimized for spectroscopy from 1 to 10keV, and the reflection gratings are optimized for the softest x-rays, 0.2 - 1keV where the performance of the calorimeter falls off.

The RGS will use dispersive spectroscopy to provide high resolution at the long wavelength end of the x-ray spectrum. In order to achieve the high spectral resolution and effective area required of the Constellation-X mission we propose using a high density, radial groove grating array in the off-plane mount. This system could potentially reach resolution ( $\lambda/\Delta\lambda$ ) of 5000 and effective area  $>3000\text{cm}^2$  as described in McEntaffer et al<sup>2</sup>.

### 1.2 Off-plane geometry

The off-plane mount at grazing incidence brings light onto the grating at a low graze angle, quasi-parallel to the direction of the grooves as shown in figure 1<sup>3,4</sup>. The light is then diffracted through an arc, forming a cone, so that this mount is also known as conical diffraction. The grating equation is now

$$\sin\alpha + \sin\beta = \frac{n\lambda}{d\sin\gamma}. \quad (1)$$

where  $d$  is the spacing between grooves.  $\gamma$  is the angle between the direction of the incoming ray and the direction of the groove at the point of impact. Light comes into the grating at an azimuthal angle of  $\alpha$  along a cone with half-angle  $\gamma$ . It is then diffracted along the same cone of half-angle  $\gamma$ , but now with an azimuthal angle of  $\beta$ .

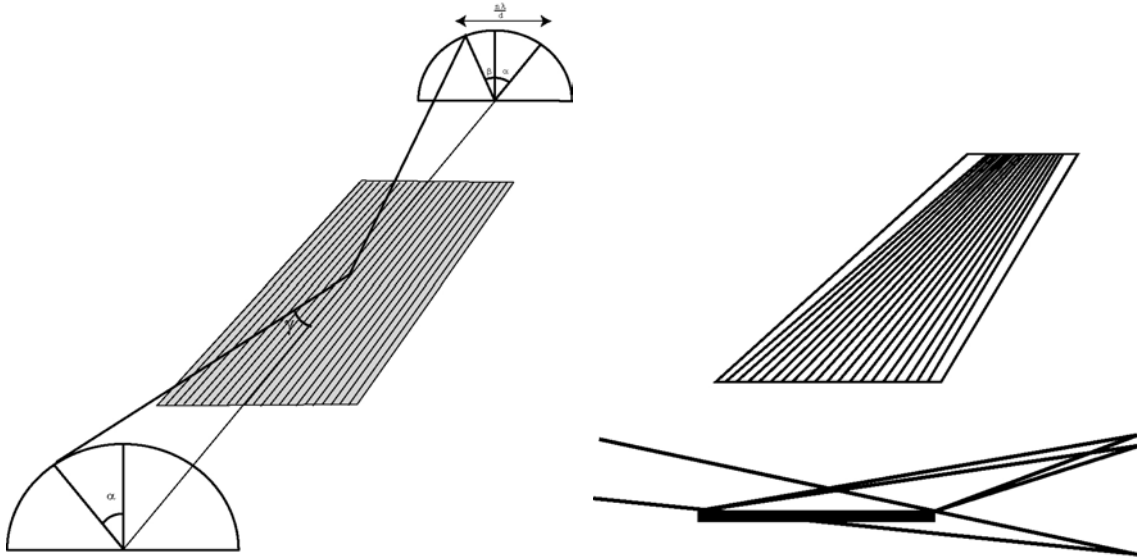


Fig. 1: Geometry of the off-plane mount. Quasi-parallel light at grazing incidence is diffracted in a cone around the direction of the rulings. Off-axis aberrations such as coma can be corrected using a radial groove grating in the converging beam.

When the gratings are placed behind the optics, the off-axis aberrations are corrected if the gratings compensate for the converging beam of the telescope. This can be done by using gratings that are ruled in a radial configuration where the grooves extend outward from a hub and match the convergence angle of the beam as shown in figure 1.

The off-plane mount supplies the natural geometry for grazing incidence reflection gratings, thus offering several advantages of which diffraction efficiency is key. The effective diffraction efficiency of the off-plane mount can be substantially higher than traditional mounts (often a factor of two) due to the groove illumination function<sup>5,6</sup>. In the off-plane mount the effect of groove shadowing is lessened. Furthermore, rigorous efficiency calculations of blazed gratings show that the off-plane mount can have efficiencies up to 70% and on average a few times higher than efficiencies obtainable with a traditional in-plane grating mount<sup>7</sup>.

## 2. Instrument Geometry

In Constellation-X the x-rays are focused by the Soft X-ray Telescope (SXT) which has a diameter of 1.6m and a 10m focal length. The SXT consists of densely packed thin shells of mirrors that provide a high efficiency x-ray image with

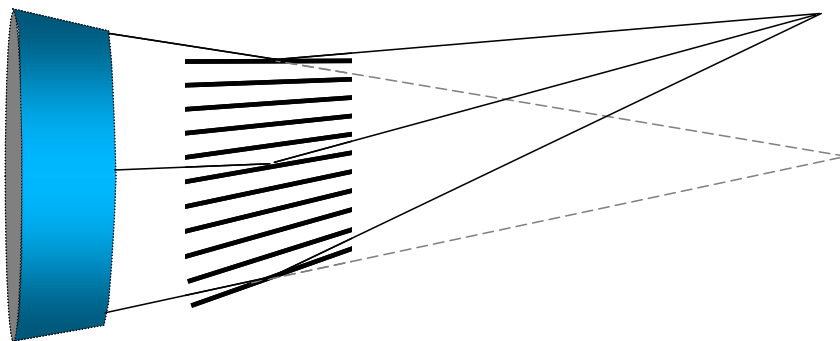


Fig 2: The gratings are placed in a fan-like array so that they can intercept the converging beam from the SXT will approximately constant graze angle across the array.

resolution of approximately 15 arcseconds. At the 10m distance this represents a focal spot of approximately  $750\mu$ .

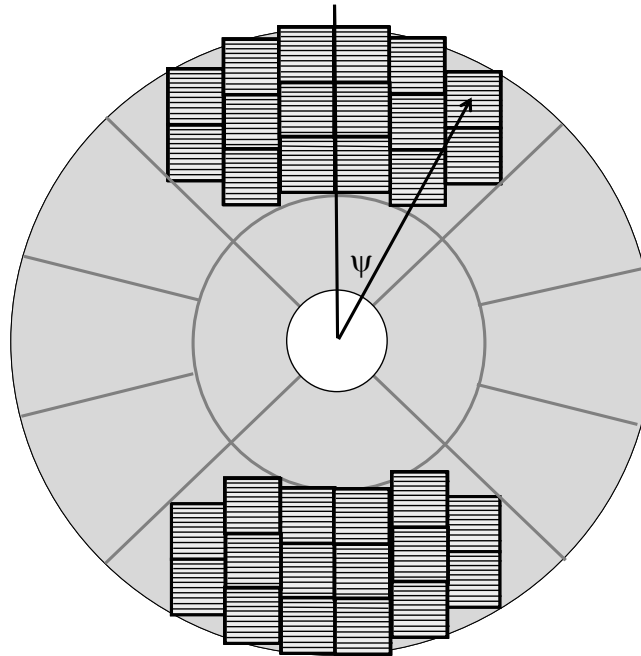


Fig 3: The gratings do not cover the entire aperture of the SXT. Here they are shown populating the regions near 6 and 12 o'clock. The angle  $\psi$  measures the position of a grating relative to the optic axis of the telescope.

The gratings are ruled on thin, planar substrates that are arranged in the converging beam behind the SXT as shown schematically in Figure 2. That allows the range of graze angles to remain approximately constant across the rear of the SXT. Since the SXT extends from 10.5 to 9.5m relative to the focus, the gratings are placed about 9.3m from the focus.

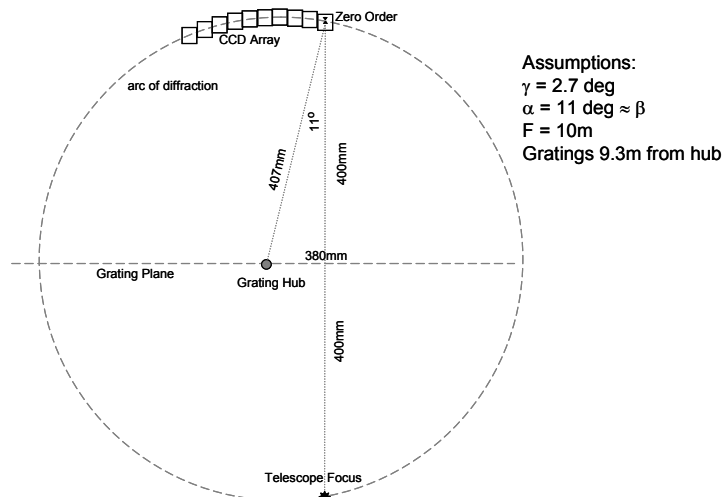


Fig 4: Geometry of the focal plane. The telescope focus is shown below. The horizontal dashed line represents the projection of the plane of the gratings onto the focal plane. The hub of the radial gratings is shown, and the arc of diffraction is a circle around the hub that passes through zero order.

The gratings do not cover the entire rear of the SXT. Instead they populate the area of the SXT where  $\psi$  is between 330 and 30 and again between 150 and 210 degrees as shown in Figure 3.  $\psi$  is defined to be zero in the direction that zero

order reflects off the gratings. This restriction of azimuthal angle allows for a substantial increase in resolution due to the subaperturing effect first described by Cash<sup>3</sup>.

The resulting focal plane geometry is shown schematically in Figure 4. The zero order is to be found about 800mm from the telescope focus, and the spectrum is found along an arc of diffraction with radius of about 400mm. An array of CCD's is mounted along the arc.

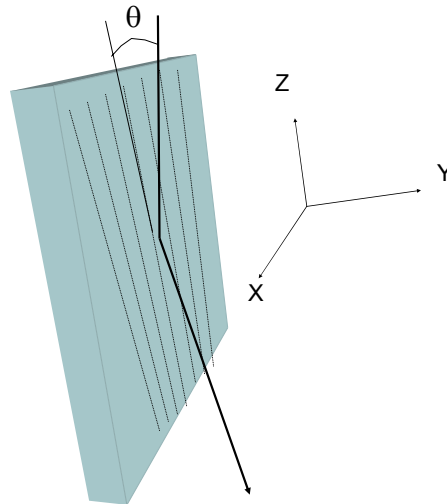


Fig 5: Definition of grating coordinates. Y is perpendicular to the plane of the grating, and x is the direction of dispersion.

In Figure 5 we show the definition of the coordinates of the grating itself. Y is the direction perpendicular to the plane of the grating. X is the direction of dispersion, and the photons travel near the negative Z direction.

## 2. Tolerance Table

We have analyzed the requirements for the quality of the grating surfaces and their orientations and present the results in Table 1. The definitions of the parameters are given in the table caption.

The table is divided into four parts. Above the line are tolerances on quality of the optical plane of the gratings. Below the line are the tolerances on the six degrees of freedom in positioning the gratings. To the left are the tolerances needed to create a zero order image of size suitable for dispersion. To the right are the tolerances necessary to support the dispersed spectrum.

Each tolerance is dependent on the variable  $\epsilon$ . We use epsilon as an approach to a tolerance budget. If  $\epsilon$  is set to one, then that tolerance will consume all of the available error. Since the tolerances tend to add in quadrature, values of  $\epsilon$  in the 0.1 to 0.3 range are usually enough to keep the system within specification. As the concept and design matures, the values of  $\epsilon$  for each tolerance can be adjusted to conform the optimum balance between importance and cost of achievement.

**Table 1: Constellation X Off-plane Grating Tolerances**

Error Type	Zero-order Allowable Tolerances			Spectrum Allowable Tolerances		
	Equation	$\epsilon$	Grating Res = 3000	Equation	$\epsilon$	Grating Res = 3000
Pitch Slope error	$\zeta < \frac{\epsilon\omega}{2}$	0.3	2.25 arcsec	$\zeta < \frac{\epsilon ds}{F \sin \psi}$ and $\zeta < \frac{\epsilon s}{F}$	0.3	0.78 arcsec
Pitch Surface error	$\delta < \frac{\epsilon\omega h}{8n}$	0.3	0.27 $\mu$ m	$\delta < \frac{\epsilon ds w}{4nF \sin \psi}$ and $\delta < \frac{\epsilon s w}{4nF}$	0.3	0.10 $\mu$ m
Roll Slope error	$\zeta < \frac{\epsilon\omega}{2 \sin \theta}$	0.3	64.5 arcsec	$\zeta < \frac{\epsilon ds}{2F \sin \theta}$	0.3	5.6 arcsec
Roll Surface error	$\delta < \frac{\epsilon\omega w}{8n \sin \theta}$	0.3	8.6 $\mu$ m	$\delta < \frac{\epsilon ds w}{8nF \sin \theta}$	0.3	0.75 $\mu$ m
$\delta_x$	$\delta_x < \epsilon w$	0.1	(up to 11mm) Set at 1mm	$\delta_x < \frac{\epsilon F \sin \theta}{R \sin \psi}$	0.3	66.3 $\mu$ m
$\delta_y$	$\delta_y < \frac{\epsilon\omega F}{2 \cos \theta}$	0.1	33.8 $\mu$ m	$\delta_y < \frac{\epsilon ds}{\sin \psi}$ and $\delta_y < \epsilon s$	0.3	36.0 $\mu$ m
$\delta_z$	$\delta_z < \epsilon h$	0.1	(up to 11mm) Set at 1mm	$\delta_z < \frac{\epsilon F}{R}$	0.3	0.95mm
$\phi_x$ (pitch)	$\phi_x < \frac{\epsilon\omega}{2}$	0.1	0.75 arcsec	$\phi_x < \frac{\epsilon ds}{F \sin \psi}$ and $\phi_x < \frac{\epsilon s}{F}$	0.3	0.78 arcsec
$\phi_y$ (yaw)	$\phi_y < \frac{2\epsilon w}{h}$	0.1	27.2 arcsec	$\phi_y < \frac{\epsilon \sin \theta}{R \sin \psi}$	0.3	1.4 arcsec
$\phi_z$ (roll)	$\phi_z < \frac{\epsilon\omega}{2 \sin \theta}$	0.1	21.5 arcsec	$\phi_z < \frac{\epsilon ds}{2F \sin \theta}$	0.3	5.6 arcsec

Where  $\delta$ =allowable linear error,  $\phi$ =allowable angular error,  $\omega$  = telescope resolution,  $\theta$  = graze angle,  $\epsilon$ =fractional limit,  $n$ =# of waves in surface error,  $w$  = grating width,  $h$ =grating height, and  $F$ =distance from gratings to detector,  $R$ =resolution ( $\lambda/d\lambda$ ),  $s$ =telescope spot size,  $ds$ =sub-apertured image width,  $\psi$ =azimuthal position of grating on telescope. Quantified tolerances assume:  $\omega=15$ arcsec,  $\theta = 2^\circ$ ,  $w=h=110$ mm,  $n=1$ ,  $F=9.5$ m,  $s=750$  $\mu$ m,

In the third column of each quadrant we evaluate the tolerance for a desired spectral resolution of 3000. As the resolution goal rises, then the value of  $ds$  decreases, and the tolerance tightens. On the whole one can see that angles need to be set to the arcsecond level, and positions to tens of microns if very high spectral resolution is to be achieved., although there are some errors to which the system is far less sensitive.

The most difficult requirement on grating quality is the pitch slope error. As one runs along the length of a groove the grating must remain flat to better than an arcsecond. This is similar to the quality needed in pointing the gratings with respect to each other. Overall, keeping the gratings flat and properly in alignment is the key to a successful RGS for Constellation-X.

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### References

1. H.D. Tananbaum, N.E. White, J. A. Bookbinder, F. E. Marshall, and F. Cordova, "Constellation X-ray mission implementation concept and science overview," *Proc. Soc. Photo-Opt. Instr. Eng.*, **3765**, 62-72, 1999.
2. R.L. McEntaffer, W. Cash and A. Shipley, "Off-plane gratings for Constellation-X", *Soc. Photo-Opt. Instr. Eng.*, **4851**, 549-556, 2002.
3. W. Cash, "X-ray optics. 2: A technique for high resolution spectroscopy", *Applied Optics*, **30**, 1749-1759, 1991.
4. R. Catura, R. Stern, W. Cash, D. Windt, J.L. Culhane, J. Lappington, K. Barnsdale, "X-ray Objective Grating Spectrograph", *Proc. Soc. Photo-Opt. Instr. Eng.*, **830**, 204-216, 1988.
5. M. Neviere, D. Maystre, W.R. Hunter, "On the use of classical and conical diffraction mountings for XUV gratings", *JOSA*, **68**, 1106-1113, 1978.
6. W. Werner, "X-ray efficiencies of blazed gratings in extreme off-plane mountings", *Applied Optics*, **16**, 2078-2080, 1977.
7. L.I. Goray, "Rigorous efficiency calculations for blazed gratings working in in- and off-plane mountings in the 5-50Å wavelength range", *Proc. Soc. Photo-Opt. Instr. Eng.*, these proceedings.